EFFECT OF VOID RATIO AND GRADATION ON SHEAR STRENGTH PARAMETERS OF GRANULAR SOILS

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ABSTRACT: This paper was mainly focused on an experimental study conducted to investigate the effects of changing grain size distribution on shear strength characteristics of sandy soils. Ravi sand, Chenab Sand and Lawrencepur sand were mixed with each other in varying proportions to constitute sand samples of varying gradations. Grain size analysis (GSA), specific gravity, index density and direct shear tests were performed to investigate the effects of changing gradation on void ratios and the shear strength parameters, respectively. Based on the experimental results of GSA and direct shear tests, multivariate regression analysis was carried out and correlation between GSA parameters and internal friction angle (ϕ) was proposed. In order to validate the proposed model, an independent set of data based on testing of thirty sand samples was used. The value of internal friction angle measured experimentally differed by about ±5% from value predicted by the proposed correlation. It was also observed that by changing the mean grain size, the void ratios first decreased and afterwards it increased. In addition to this, direct shear tests were also performed on sand samples by adding non plastic fines up to 30% in three basic sands to examine their effect on shear strength parameters of sandy soils. The friction angle decreased as fine content increased where as the apparent cohesion increased with the increase in the fines within the range of fine contents investigated.

Keywords: Relative density, internal friction angle, co-efficient of uniformity, co-efficient of curvature, maximum dry unit weight, void ratios

INTRODUCTION

Shear strength of any soil deposit is a function of two parameters, i.e., cohesion (c) and angle of internal friction (ϕ). However, in general engineering practice, the shear strength of cohesive soil is based on only cohesion (c), whereas, in case of cohesionless soil, the shear strength is mainly based on angle of internal friction. Design of any earthen structure is not possible without the knowledge of these parameters. Generally, in developed countries, all necessary engineering properties regarding the material encountered/ used in any project are properly documented and correlations among various parameters are developed for quick evaluations in similar future projects. However, in Pakistan, there is not much work in this regard. This research is focused in predicting shear strength parameters of sand on the basis of various grain size distribution parameters. Various researchers including (Becker et al. 1972, Chan and Page 1997, Santamarina and Cascante 1998, Madhav and Arlekar 2000, Hara et al. 2004, Liu and Evett 2005 and Igwe et al. 2006 have worked in this area.

(Becker et al. 1972) depicted that relative density is the major contributor in shear strength and ϕ increases with the increase in relative density. (Chen and Page 1997) plotted internal friction angle with roundness and found that frictional resistance decreased with roundness. Stress strain trends documented by (Santamarina and Cascante 1998) suggested that internal friction angle increased with the surface roughness of the particles. (Madhav and Arlekar 2000) conducted series of shear test on sandy soils at varying densities. They concluded that rate of dilation increase with relative density. (Hara et al. 2004) performed undrained cyclic triaxial tests on river sands and decomposed granite soil sample having different relative densities and uniformity coefficient. According to (Liu and Evett 2005), sand is free draining material. Thus, water in the void spaces would drain out quickly causing the shear strength to remain constant. (Igwe et al. 2006) conducted research to understand the liquefaction potential of sands. They performed ring shear test on well graded, intermediately graded and narrowly graded sand and concluded that intermediately graded and narrowly graded show more swell potential than well graded samples. They concluded that undrained strength of well graded samples will always be more than poorly graded samples provided well graded samples do not contain crushable particles. (Kara et al. 2013) performed direct shear tests on marine sand and concluded that friction angle of sand increases with the increase in grain size. (Meyerhof, 1956) suggested that angle of internal friction can be estimated from known value of relative density (D_r) Eq. (1) can be used for estimation of ϕ of granular soil with more than 5% fines and Eq. (2) with less than 5% fines.

$\phi = 25 + 0.15D_r$ (1)

$\phi = 30 + 0.15D_r$ (2)

(Alim et al, 2013) studied the effect of fine material on the shear strength characteristics of granular soils. Their research reveals that both cohesion and angle of internal friction increase with the increase in fines up to 20%. Also, maximum and minimum void ratio depends upon the size, shape angularity, grain size distribution along with the test procedure adopted. (Cubrinovski and Ishihara 2002) proposed a correlation between e_{max} and e_{min} for sands as given by Eq. (3) and Eq. (4).

$e_{max} = 0.072 + 1.53e_{min}$ (for sand with 0 to 5% fines) (3)and $e_{max} = 0.25 + 1.37e_{min}$ (for sand with 5 to 15% fines) (4)

Similarly, (Gomaa and Abdelrahman 2007) has proposed that $e_{min} = 0.57e_{max}$ with $R^2 = 0.75$.

MATERIALS AND METHODS

The sand samples used in this research were prepared by mixing local sand (i.e., Ravi, Chenab and Lawrencepur sands) in varying proportions to generate different grading of sand samples. The proportion of Ravi sand in all the samples was between 50-100%, while proportion of Chenab and Lawrencepur sand varied between 10-40%. The reconstituted sand samples were subjected to the following tests.

a) Grain size analysis (ASTM D-422)
b) Specific Gravity of the soil solids (ASTM D-854)
c) Soil Classification (ASTM D-2487)
d) Maximum Index Density Test (ASTM D-4253)
e) Minimum Index Density Test (ASTM D-4254)
f) Direct Shear Test (ASTM D 3080)

The void ratios corresponding to maximum index density $(\gamma_{d\max})$ and minimum index density (γ_{dmin}) determined through vibratory table following ASTM D-4253 and ASTM D-4254 were calculated as

$$e_{max} = \left(\frac{G_s\gamma_w}{\gamma_{d\ min}}\right) - 1 \quad (5)$$

and

$$e_{min} = \left(\frac{G_s\gamma_w}{\gamma_{d\ max}}\right) - 1 \quad (6)$$

The grain size distribution curves of test samples were plotted in Fig. 1 whereas the results of the other tests listed above are summarized in Table 1.

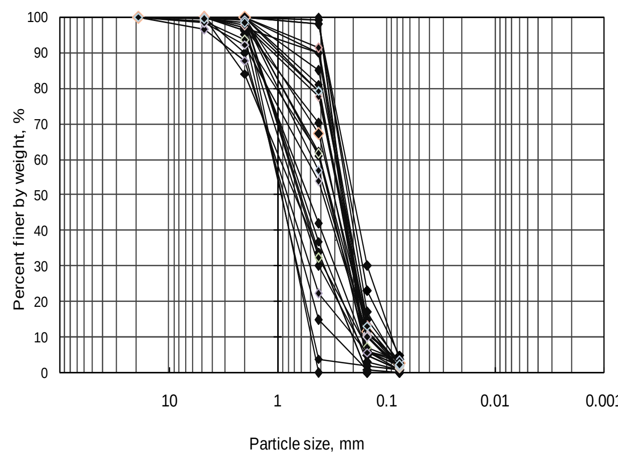

Fig. 1 Grain size distribution curves of sand samples

RESULTS AND DISCUSSION

- Fig. 1 plots the results of grain size analysis of all the samples used in this research. It can be observed from the figure that all the selected samples fall in the range of medium to fine sand. Mean grain size (D_{50}) of all the samples was in the range of 0.9 mm to 0.20 mm and effective grain size (D_{10}) was in the range of 0.50 mm to 0.08 mm. The coefficient of uniformity (C_u) varied from 2.20 to 2.33 and the coefficient of curvature (C_c) was within 0.89 to 1.19.
- Table 1 summarized the results of grain size analysis, specific gravity and soil classification. Table 2 which contained the results of index density tests and direct shear tests. The results of first 20 samples were used for the development of correlations where as the results of remaining 10 samples as indicated in the same tables were used for the validation of correlations. The samples used in the study contained sand (material passing from US # 4 and retained on US # 200 sieve) varying between 95 and 100 %. No gravel fraction (percent retained on US # 4 sieve) was present in the samples, whereas the fine contents (percent passing US # 200 sieve) varied from 0 to 4%.
- The specific gravity of the tested samples falls in the range of 2.62 ~ 2.73. All the soil samples were poorly graded sand (SP) as classified by Unified Soil Classification System (USCS) described in ASTM D-2487. Maximum and minimum void ratios $(e_{max}\ and\ e_{min})$ determined through Eq. (3) and Eq. (4), respectively, have been summarized in Table 2. The value of e_{max} varied from 0.67 to 1.04 and e_{min} from 0.74 to 0.38, respectively. Direct shear tests were performed according to ASTM D 3080 on all the samples under dry condition at relative densities of 50%, 75% and 95%. The results of direct shear have been summarized in Table 2. The value of □□varies from 26.7^o to 36.9^o with no cohesion.

DEVELOPMENT OF CORRELAATION

Correlation between ϕ and grain size parameters

Experimental data was divided into independent and dependent variables. Independent variables consisted of parameters related to grain size distribution which included mean grain size (D_{50}), effective grain size (D_{10}), uniformity coefficient (C_u), curvature coefficient (C_c), fine contents (F_{200}) and dry unit weight (γ_d) of the remolded samples. Internal friction angle (ϕ) was considered as dependent variable. By performing backward regression analysis, the potential independent variables were identified which affected the dependent variable. Based on regression analysis, mean grain size (D_{50}), curvature coefficient (C_c) and dry unit weight (□$_d$) were selected as three independent variables which significantly effected the internal friction angle (ϕ).

Table 1 Summary of Grain Size Analysis and Specific Gravity Tests on samples

Sample No.	Gravel (%)	Sand (%)	Silt & Clay (%)	D_{50}	Cu	Cc	Gs	USCS Classification
1	0	96	4	0.21	2.78	1.28	2.69	SP
2	0	97	3	0.22	2.25	1.11	2.69	SP
3	0	100	0	0.25	1.56	0.88	2.69	SP
4	0	100	0	0.26	1.71	0.81	2.67	SP
5	0	99	1	0.29	2.82	0.84	2.67	SP
6	0	98	2	0.3	2.11	0.71	2.67	SP
7	0	97	3	0.37	2.39	1.01	2.66	SP
8	0	96	4	0.45	3.39	0.71	2.66	SP
9	0	100	0	0.64	3.81	0.91	2.66	SP
10	0	100	0	0.73	4.76	0.80	2.66	SP
11	0	100	0	0.9	2.20	0.87	2.66	SP
12	0	96	4	0.2	2.56	1.24	2.69	SP
13	0	99	1	0.35	2.22	0.80	2.70	SP
14	0	99	1	0.6	4.20	1.19	2.72	SP
15	0	98	2	0.21	2.36	1.38	2.63	SP
16	0	99	1	0.26	1.81	0.88	2.67	SP
17	0	99	1	0.7	4.86	0.93	2.65	SP
18	0	99	1	0.27	2.12	0.83	2.66	SP
19	0	99	1	0.8	2.00	0.90	2.68	SP
20	0	100	0	0.7	1.9	0.94	2.69	SP
21	0	99	1	0.25	1.87	0.95	2.72	SP
22	0	96	4	0.65	4.20	1.05	2.65	SP
23	0	98	2	0.8	4.00	1.44	2.73	SP
24	0	98	2	0.28	2.00	1.40	2.69	SP
25	0	97	3	0.3	2.38	0.73	2.66	SP
26	0	97	3	0.38	3.1	0.79	2.66	SP
27	0	99	1	0.28	2	0.98	2.62	SP
28	0	99	1	0.3	2.5	0.9	2.69	SP
29	0	98	2	0.4	3.12	0.78	2.64	SP
30	0	98	2	0.29	2	0.89	2.68	SP

D_{50} – Mean Diameter, ***C_u*** – Uniformity coefficient, ***C_c***- Curvature coefficient, ***G_s***- specific gravity of soil solids

Sample No.	Index Density Test		Direct Shear Test		Direct Shear Test		Direct Shear Test	
			Dr (50%)		Dr (75%)		Dr (95%)	
	Maximum Void Ratio e_{max}	Minimum Void Ratio e_{min}	c (kPa)	ϕ (deg)	c (kPa)	ϕ (deg)	c (kPa)	ϕ (deg)
1	1.04	0.74	0.0	29.86	0.0	31.34	0.0	31.38
2	1.02	0.73	0.0	31.38	0.0	32.7	0.0	33.42
3	0.99	0.68	0.0	32.45	0.0	33.18	0.0	34.01
4	0.96	0.64	0.0	32.57	0.0	32.94	0.0	33.86
5	0.92	0.60	0.0	32.29	0.0	32.82	0.0	33.62
6	0.91	0.59	0.0	32.71	0.0	33.55	0.0	34.36
7	0.84	0.53	0.0	35.04	0.0	35.26	0.0	35.6
8	0.8	0.50	0.0	34.01	0.0	35.14	0.0	36.54
9	0.86	0.56	0.0	35.17	0.0	35.97	0.0	36.71
10	0.89	0.57	0.0	35.21	0.0	36.15	0.0	36.9
11	0.91	0.59	0.0	35.14	0.0	36.5	0.0	36.97
12	1.04	0.73	0.0	29	0.0	31	0.0	31.7
13	0.9	0.52	0.0	33.4	0.0	33.9	0.0	34.2
14	0.85	0.54	0.0	35	0.0	36.2	0.0	36.8
15	0.96	0.65	0.0	26.67	0.0	28.4	0.0	30.4
16	0.85	0.52	0.0	29.63	0.0	30.91	0.0	31.64
17	0.95	0.60	0.0	32.4	0.0	33.9	0.0	36.1
18	0.9	0.56	0.0	29.82	0.0	31.8	0.0	34.03
19	0.93	0.63	0.0	33.1	0.0	34.8	0.0	35.6
20	0.95	0.68	0.0	35.1	0.0	35.9	0.0	36.4
21	0.95	0.61	0.0	30.1	0.0	32	0.0	32.4
22	0.89	0.55	0.0	31	0.0	33.4	0.0	34.68
23	0.68	0.38	0.0	32	0.0	33.8	0.0	35.4
24	0.88	0.55	0.0	30.5	0.0	31.4	0.0	32.2
25	0.87	0.53	0.0	31.5	0.0	32.5	0.0	33.4
26	0.86	0.64	0.0	31.6	0.0	31.8	0.0	33.6
27	0.84	0.62	0.0	32.6	0.0	33	0.0	34.6
28	0.78	0.58	0.0	30.7	0.0	31.8	0.0	33.8
29	0.67	0.48	0.0	33	0.0	33.4	0.0	35.8
30	0.86	0.59	0.0	30.6	0.0	32.6	0.0	33

Development of Model: The Statistical Product and Service Solution (SPSS) computer program was used to analyze the data presented in Table 1 and Table 2. As indicated by regression analysis, mean grain size (D_{50}), curvature coefficient (C_c) and dry unit weight (γ_d) have a significant effect on internal friction angle of sandy soil as compared with other input parameters. Therefore, only these three variables were used in correlation. Regression analysis was used to calibrate the correlation and the values of coefficients for input and output were calculated. Based on the analysis, the following correlation is being proposed as given by Eq. 7.

$$\phi = 11.1 + 6.54 D_{50} + 1.48 \gamma_d - 3.73 C_c \quad (7)$$

The results of the regression analysis by SPSS software revealed that standard error of estimate (SEE) is 0.5 which indicated a good agreement between experimental and predicted values. The correlation coefficient (R) for Eq. (7) was 0.9 and was regarded as good correlation coefficient in geotechnical engineering. Analysis of variance (ANOVA) was carried out by SPSS software to determine F- statistic for output parameters and *t*-

statistics for input parameters. The model F value for ϕ was greater than critical F indicating that Eq. (7) was significant. Similarly, absolute t- statistics for input parameters was greater than t- significance of the correlation model indicating that C_c, D_{50} and (γ_d) passes t-test suggesting that input parameters were significant and a slight change in their values had considerable effect on output parameters ϕ □

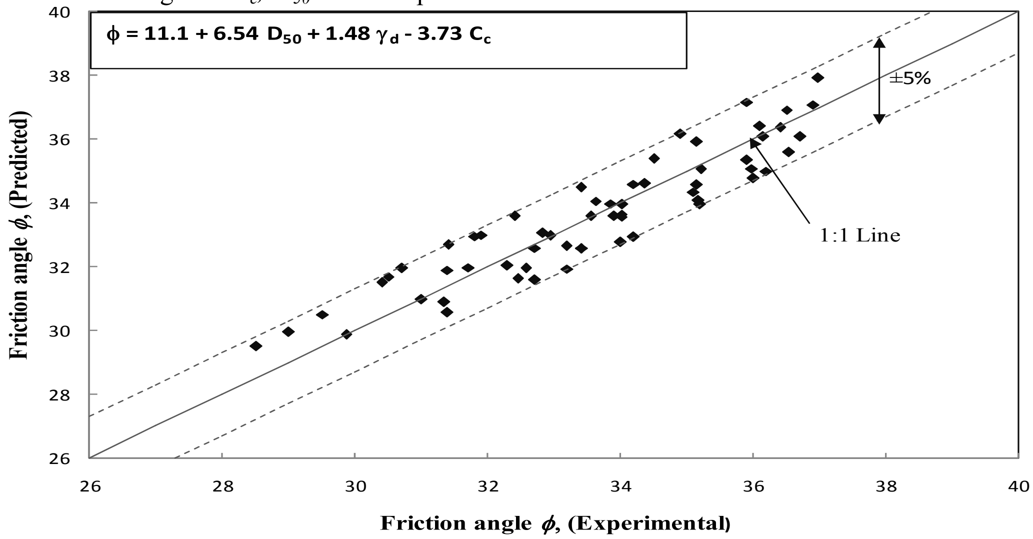

Fig. 2 Experimental versus predicted values of Friction angle

Validation of Model: The validation of the correlation was performed by using test results of a new set of thirty sand samples not used previously in the development of predictive model. Experimental values of □ were plotted against the predicted values by the Eq. (7) and are shown in Figure 3. The predicted values of ϕ □□fall within ± 5% of the measured ϕ □values by direct shear test. The empirical relationships developed by (Meyerhof 1956) were also used to predict internal friction angle for these samples. For ϕ, 13 out of 15 predictions were exceeding the limits of ± 5% by using (Meyerhof 1956) equation. It appears that Eq. (5) cannot be used for pure sand containing non-plastic fines whereas the Eq. (7) proposed by present research can be effectively used for such type of soils.

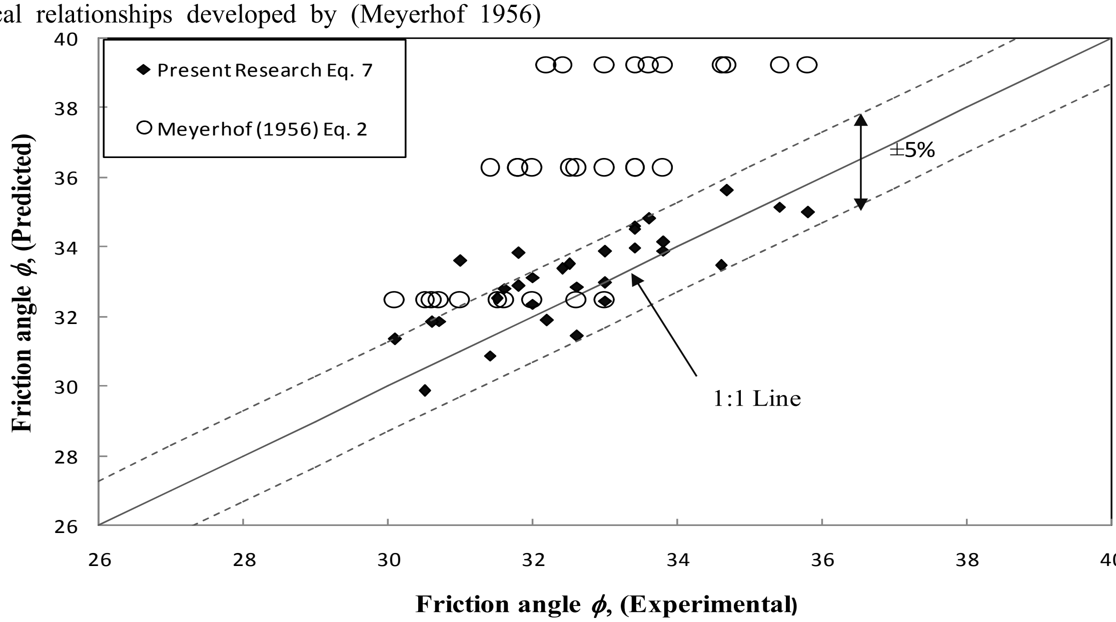

Fig. 3 Validation of Friction angle

Correlation between Maximum and Minimum Void Ratios: The results of maximum and minimum void ratio from Table 2 are presented in Fig. 4. As evident from Fig. 4, there was a linear relationship between maximum and minimum void ratio which is expressed by Eq. (8) as below

$$e_{max} = 0.853 e_{min} + 0.404 \quad (8)$$

Standard error of estimate for Eq. (8) was 0.02, F–statistics for output parameter was 176.75 and t-statistics for input parameters was 10.30 and 13.26, respectively. These regression statistics indicated that the correlation between maximum and minimum void ratio was significant. The lower and upper limit of intercept at 95% confidence interval was 0.322 ~ 0.487 and that of e_{min} was between 0.718 ~ 0.988. The coefficient of determination (R^2) for Eq. (8) is 0.91 indicating that only 9% of variation in the values of e_{max} was not accounted by regression.

The results of Eq. (8) have been compared with Eq. (3) by predicting the values of e_{min} by using both the equations based on e_{max} data in Table 2 and the results have been shown in Fig. 4. As evident from Fig. 4, both the equations were in good agreement at $e_{min} = 0.5 \sim 0.6$ and gave a difference of 10% when e_{min} was equal to 0.73. The probable reason for this difference in prediction by both the equations may be that a different standard of determining e_{min} has been followed, i.e., Eq (3) and Eq. (4) was based on Japanese standards, whereas the Eq. (8) was based on ASTM procedure. Therefore, caution was needed when applying Eq. (3)/ Eq. (4) and Eq. (8) to estimate e_{min} or e_{max}.

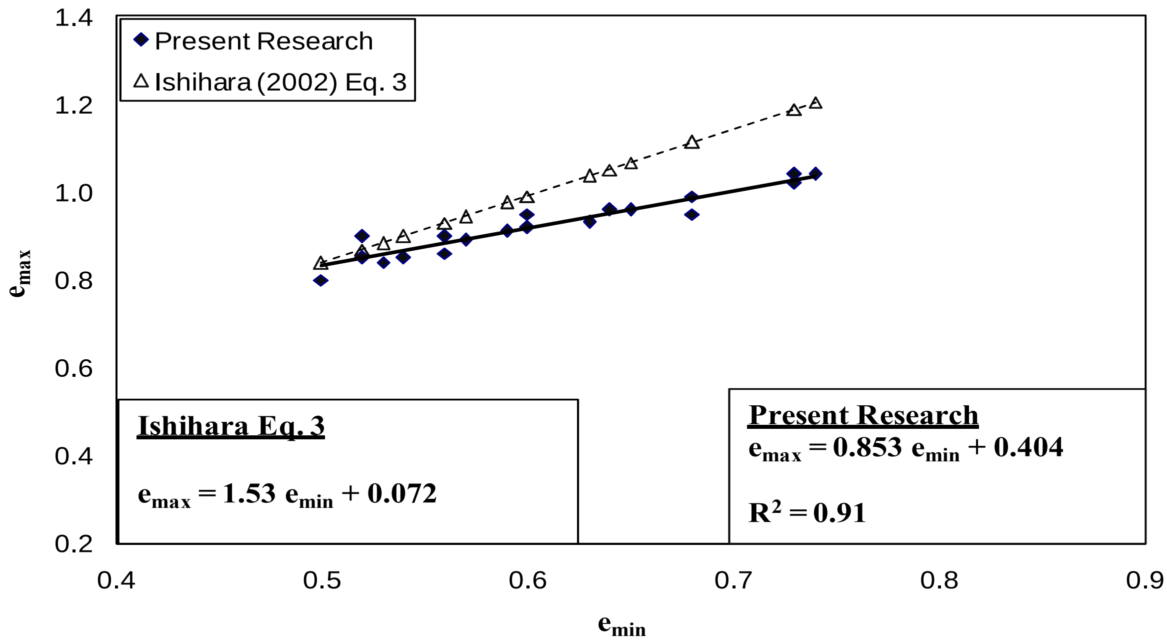

Fig. 4 Minimum and Maximum void ratio

Effect of grain size on void ratios

Fig. 5 explained the effect of changing gradation, in terms of D_{50}, on maximum and minimum void ratios (e_{max} *and* e_{min}). It is clear from Fig. 5 that for initial portion of the curve i.e., when mean grain size was between 0.1 to 0.4 mm, there was decrease in void ratio. However, when D_{50} crosses 0.5 mm, the void ratios again started increasing. Therefore, it can be concluded that for a given vibration, the sand attain their maximum packing corresponding to mean grain size in the range of 0.4~0.5 mm. The reason for this type of behavior was that when difference between mean grain size and effective grain size was small, void ratio was minimum. However, when this difference increased void ratio also increased. This observation was in line with Cho et al. (2006). They concluded that due to angularity volume of voids start increasing after a critical grain size i.e 0.4- 0.5 mm.

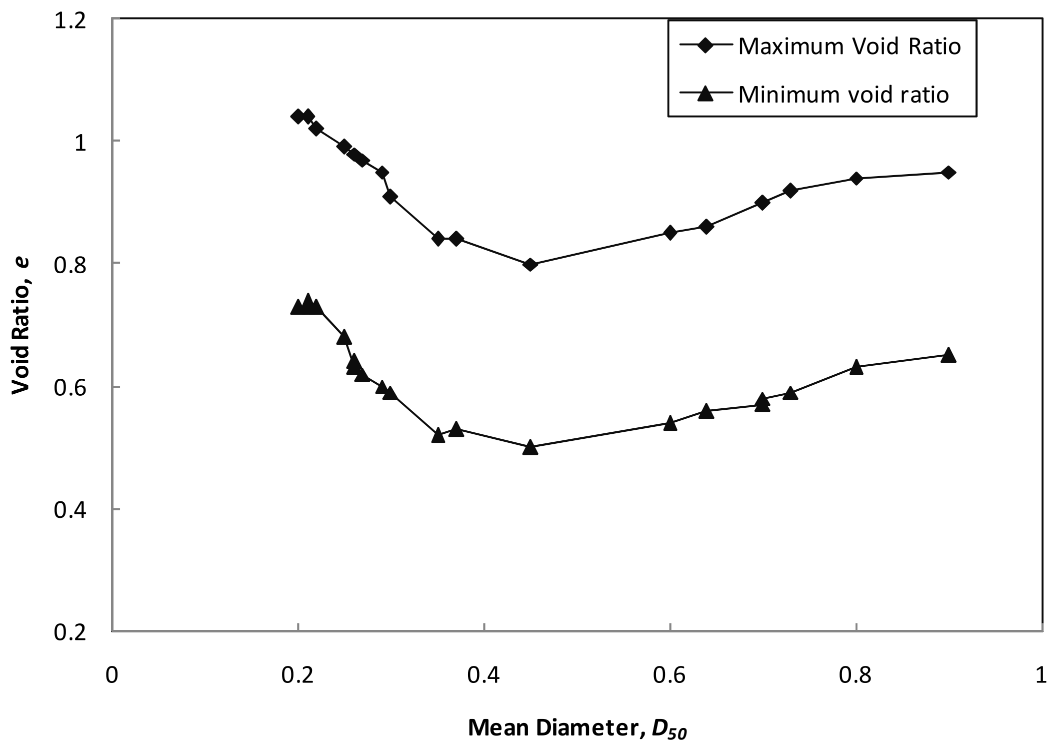

Fig. 5 Mean grain size versus void ratio

Effect Of Fines On Shear Strength Parameters: In order to investigate the effect of varying fine contents (percent passing US # 200 sieve) on density (void ratio) and shear strength characteristics of sandy soil, samples of three locally available sand, i.e., Ravi sand, Chenab sand and Lawrencepur sand were procured and then by mixing non plastic fines in them, sand samples with fine contents varying from 0% to 30% were generated. Figure 6 showed the gradation curves of three basic sand samples along with reconstituted samples with varying fine contents. Further, on these samples, direct shear tests were performed. The following section described the results of direct shear tests.

Effects of fines on shear strength parameters (c &): Reconstituted samples were prepared by mixing non plastic fines in Ravi, Chenab and Lawrencepur sand at varying proportion i.e 5%, 10%, 20% and 30%. Direct shear test in dry condition along with sieve analysis was carried out on these reconstituted samples. It was observed that as the amount of fines increased the value of internal friction angle decreased. This trend was presented graphically in Fig. 7. The decrease in internal friction angle was 28.8% in Ravi sand, 21.8 % in Chenab sand and 18.2% in Lawrencepur sand. This can be concluded that for fine sand percent decrease was more as compared to coarse sands. Secondly, it was also observed that for fine fraction varying between 20% to 30 % reduction in internal friction angle was only 4.5% as compared to sample having 5 or 10%. Fig. 8 represented the effect of fines on apparent cohesion. It can be inferred from the figure that apparent cohesion increased with the increase in fines percentage. Increase in apparent cohesion was more prominent in Ravi sand as compared to Lawrencepur sand. However for pure sand, the value of apparent cohesion was zero and it increased by about 10% when fines were increased from 5 to 30%.

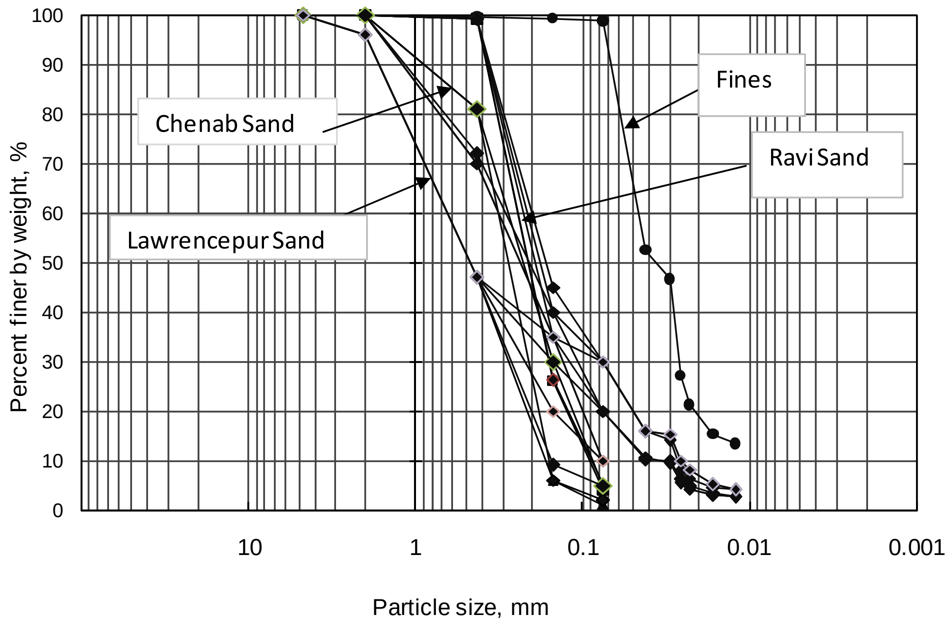

Fig. 6 Grain size distribution curves of Ravi, Chenab, Lawrencepur sand and Fines

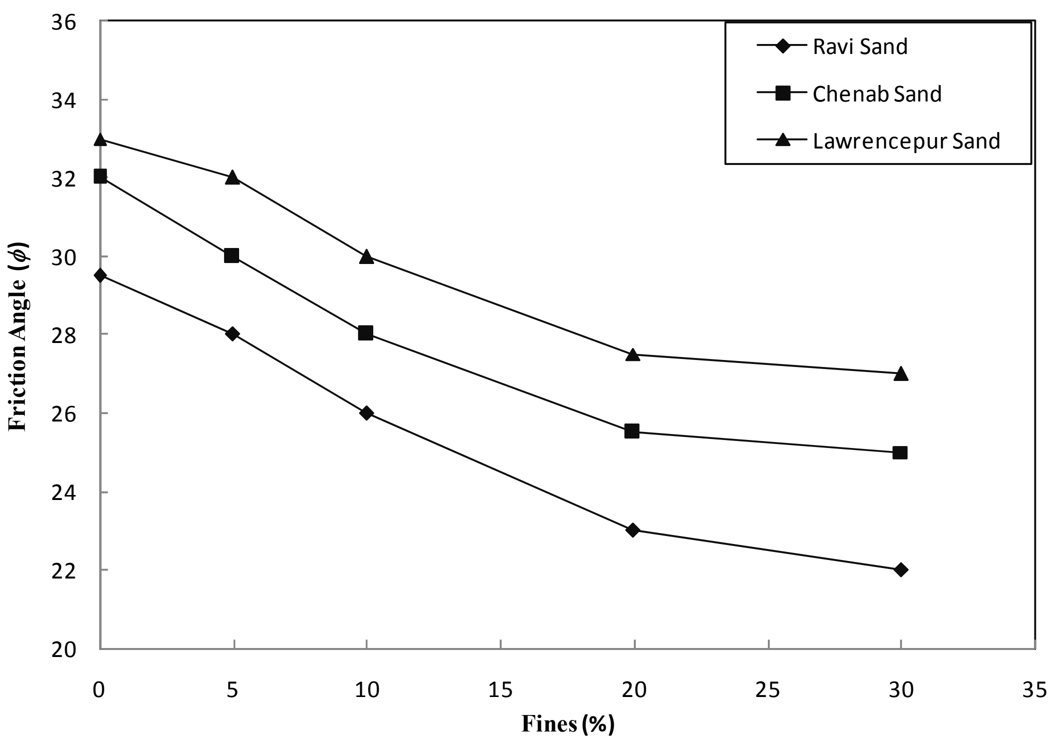

Fig. 7 Fines versus Friction angle

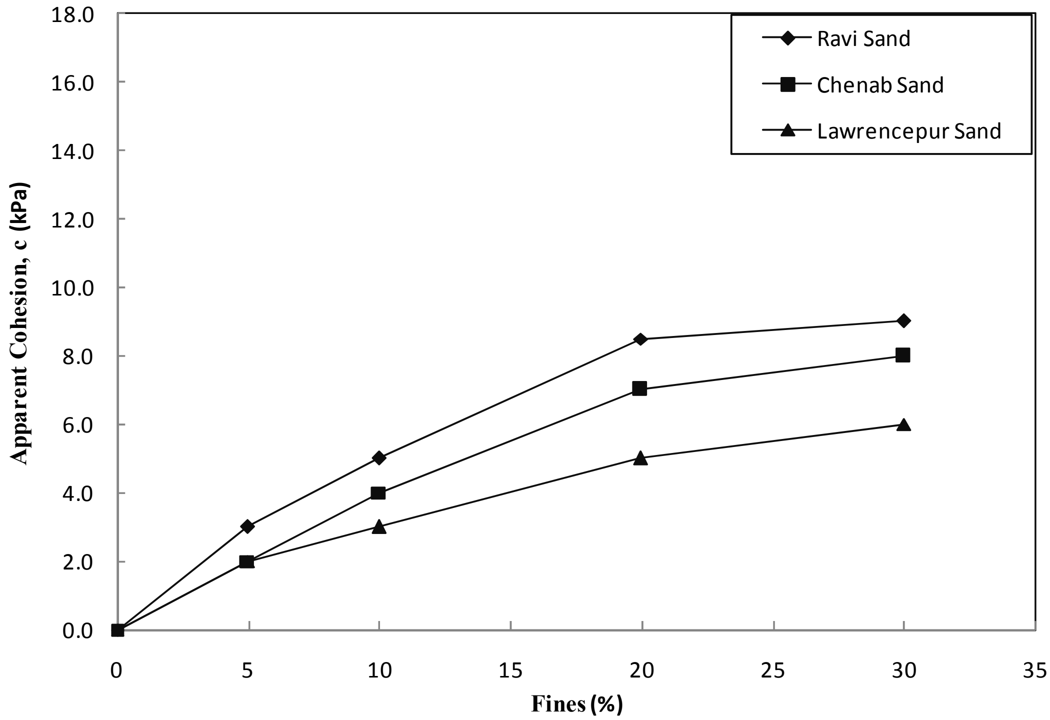

Fig. 8 Fines versus Apparent Cohesion

Conclusion: In this study, the results of classification tests, index density tests and direct shear tests performed on thirty (30) sandy samples have been presented. Based on the experimental data, correlation of internal friction angle (ϕ) with dry unit weight (γ_d), mean grain size (D_{50}), and coefficient of curvature (C_c) have been proposed. Further, the effects of increase in fines on shear strength parameters (c & ϕ) of sandy soil have also been investigated. Based on the results of the study, the following conclusions are drawn:

- The internal friction angle (ϕ) can be predicted based on mean grain size (D_{50}), coefficient of curvature (C_c) and dry unit weight (γ_d) using the relation: $\phi = 11.1 + 6.54 \times D_{50} + 1.48 \times \gamma_d - 3.73 \times C_c$ The experimental versus predicted values of ϕ fall within ± 5% indicating good prediction accuracy of the model.
- There was a linear relationship between maximum and minimum void ratio of sandy soil. The minimum void ratio can be estimated by using the correlation: $e_{\max} = 0.853 \times e_{\min} + 0.404$
- With the increase in mean grain size (D_{50}) up to 0.4~0.5 mm, there was decrease in void ratio and afterwards, the void ratio increased with the increase in mean grain size.
- The internal friction angle (ϕ) obtained through direct shear test decreased with the increase in non-plastic fines up to 30%. However, it can be concluded that decrease in internal friction angle was more significant when the fines are added upto 20 % and afterwards, the decrease in friction angle value was less significant, i.e., within 4.5%.
- Based on the results of direct shear, it can be inferred that apparent cohesion increased with the addition of fines, whereas the value of apparent cohesion was zero when the fines were lesser than 5%. However, the addition of fines up to 30%, resulted in the development of apparent cohesion in the order of 6~9 kPa.

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