RELATION BETWEEN UNIAXIAL COMPRESSIVE STRENGTH, POINT LOAD INDEX AND SONIC WAVE VELOCITY FOR DOLERITE

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ABSTRACT: Uniaxial compressive strength (UCS) of intact rocks is a major consideration in rock mechanics study for civil and mining projects. UCS direct testing is expensive, time consuming and it involves preparation of rock samples. Indirect tests are relatively cheaper, faster, and are convenient to perform in the laboratory and at site. This work presents the development of possible empirical relations between UCS values determined by direct and indirect testing methods including Point Load Index and Sonic Velocity for dolerite. A review of various correlations established for UCS versus point load index and UCS versus sonic wave velocity for different rocks by various researchers have also been presented.

Key words: Uniaxial compressive strength, Sonic velocity, Point load index, Dolerite, Hadda formation

INTRODUCTION

A complete site characterization for mining and geotechnical engineering applications require extensive database of geo-mechanical engineering properties of rocks. Initial assessment of site portrayal requires quick information of rockmass properties. Various constraints including economics, remote locations and time limitations restrict rock engineers to directly attain the explicit design parameters of interest. Therefore, alternative ways based on empirical or theoretical relations for estimating various geo-mechanical and physical properties of rocks are required.

Geo-mechanical characteristics of rocks are crucial for geotechnical design of subsurface structures. In mines uniaxial compressive strength (UCS) of rock mass is a key design aspect for stope and pillar, roof support, excavations in rock burst prone ground, squeezing and swelling ground etc. Determination of UCS for different rocks in laboratory is a common practice. Standard testing procedures have been established by the American Society for Testing and Materials (ASTM) and the International Society of Rock Mechanics (ISRM). However these methods are expensive, time consuming and involve high quality sample preparation (Fener et al; 2005). On the other hand, indirect testing techniques such as point load test and sonic pulse velocity for the estimation of UCS of rocks require little or no sample preparation, very less testing time and can be easily performed on-site. This study aims at to establish possible correlations between the UCS values determined by direct and indirect methods for the dolerite rock samples from Hadda Formation of Sillanwali area in Punjab province of Pakistan. The Sillanwali region is expected to host a few civil works and mining projects in future.

Point Load Index: The point load index (PLI) is extensively used for the indirect estimation of UCS. It can furnish similar data at a lower cost. The testing apparatus is portable and can be used on-site. The rock sample is compressed between conical points until failure occurs. Failure pressure is recorded and point load index Is₍₅₀₎ is determined by relation given as (1):

$$Is_{(50)} = P/De^2$$
----- (1)

Where P =failure load in lbs

De = equivalent core diameter (inches)

Several researchers have recommended a variety of empirical relations for the calculation of UCS from $I_{S(50)}$.

D'Andrea et al; (1964) proposed the following linear correlation equation for estimating UCS from $I_{S(50)}$:

$$q_u = 16.3 + 15.3 I_{s (50)}$$

where

 q_u = Uniaxial compressive strength of the rock.

 $I_{s (50)}$ = Point load index for rock core diameter of 50 mm. Broch and Franklin (1972) disclosed that UCS of rock core of NX size is roughly about 24 times its point load index and suggested following relationship between UCS and $I_{S(50)}$:

$$UCS = 24 I_{s(50)} ----- (3)$$

Bieniawski (1975) recommended the following relationship amongst UCS, PLI (I_{s} (50)) and rock core diameter (D):

UCS = $(14 + 0.175 \text{ D}) I_{s (50)}$ ----- (4)

Cargill and Shakoor (1990) conducted standard UCS and point load tests on fourteen rock types including Sandstone, Limestone, Dolomite, Marble and Syenitic Gneiss, to determine possible correlation between UCS and PLI, and established following linear relationship:

$$q_u = 13 + 23 l_{s(50)}$$
----- (5)

Grasso et al; (1992) working on calcareous mudstone core samples reported the following correlations between UCS (C_o) and PLI:

For power-fit relationship:

 $C_o = 25.67 (I_{s(50)})^{0.57}$ ----- (6) For linear-fit relationship:

 $C_0 = (9.30) I_{s(50)} + 20.04 -----(7)$

Investigations carried out by Rusnak and Mark (2000) on different rock types resulted in two distinct correlations between UCS and PLI $(I_{s(50)})$ which is as follows:

For coal measure rocks:

 $q_u = 23.62 I_{s(50)} - 2.69 - (8)$

For other different rock types:

 $q_u = 8.41 I_{s(50)} + 9.51 ----- (9)$

Quane and Russel (2003) reported the following correlations between UCS and Is(50) for strong rocks and weak rocks respectively:

 $q_u = 24.4 I_{s(50)} ----- (10)$

and

 $q_u = 3.86 (I_{s(50)})^2 + 5.65 I_{s(50)} ----- (11)$

Palchik and Hatzor (2004) performed tests on porous chalks and proposed following correlation between UCS and point load index:

 $q_u = I_s k_1 e^{-k2 n} - (12)$

where, K₁ and K₂ are empirical coefficients and n is the porosity of the sample.

Fener et al; (2005) examined the test results of eleven rock types including igneous, metamorphic and sedimentary rocks to evaluate the possibility of existence of correlation between UCS and PLI. They reported the following linear relationship:

 $q_u = 9.08 I_s + 39.32 \dots (13)$

Akram and Abu Bakar (2007) carried out UCS and point load tests on nine different rock types belonging to the Salt Range area of Punjab province in Pakistan. The test results confirmed the existence of correlation between UCS and Is(50) for two rock groups. The first group (group A) contained hard sandstones of Jutana, Baghanwala and Khewra formations along with Sakesar limestone, Khewra dolomite and Baghanwala siltstone. The second group (group B) consisted of comparatively soft rocks comprising nodular Sakessar limestone, Dandot sandstone and marl. The cited authors proposed the following correlation equations:

For rocks of group A:

UCS = $22.7921 I_{S(50)} + 13.295 - (14)$ For rocks of group B:

UCS = $11.076 I_{S(50)}$ ----- (15)

Jabbar (2011) performed an experimental study on rock samples collected from Tag Dam project and recommended the following correlation between UCS and PLI:

UCS =
$$10022.2 I_{S(50)}$$
 (KPa) ----- (16)

Kurtulus et al; (2011) performed laboratory tests on serpentinized ultrabasic cylindrical rock samples for establishing correlation between UCS and $I_{S(50)}$. They

tested twenty specimens along foliation and twenty specimens across foliation for determining both UCS and PLI.

For tests across foliation:

UCS = $15.248 I_{s(50)} - 2.2964 - (17)$

For tests along foliation:

UCS = $14.458 I_{s(50)} + 0.3852 - (18)$

Sonic Wave Velocity: Ultrasonic techniques being nondestructive and easy to apply are widely utilized for indirect estimation of mechanical and geo-physical properties of rocks. Sonic wave velocity has a direct relationship with strength of material. Various researchers have determined a close relationship between sonic wave velocity of a rock mass and rock compressive strength.

Inoue and Ohomi (1981) reported the following correlation among UCS, sonic velocity and rock density: UCS = k $\rho V_p^2 + 31.18$ ----- (19)

k= Empirical coefficient

 ρ = Density of rock

The testing program of Grasso et al; (1992) on calcareous mudstone core samples collected from five boreholes drilled for geotechnical investigations of a road tunnel in Central Italy produced following correlation linking UCS and compression wave velocity:

$$C_{o} = \exp \left[1.04 + 1.14 \text{ x } 10^{-3} \text{ (V}_{p})\right] ---- (20)$$

where

C_o = Unconfined compressive strength

 $V_p = Compression$ wave velocity

Kahraman (2001) reported the following power relationship:

 $UCS = 9.95V_p^{1.21}$ ----- (21)

Yasar and Erdogan (2004) found following simple mathematical relation between P-wave velocity and UCS for carbonate rocks:

SV = 0.0317 σ_c + 2.0195 ----- (22)

where

and

 σ_c = Uniaxial compressive strength.

SV = Sonic velocity

The investigations of Entwisle et al; (2005) produced the following exponential and power relationships between V_p and UCS respectively:

$$UCS = 0.783 e^{0.882 v_p} ----- (23)$$

$$UCS = 0.292 V_{p}^{4.79} ----- (24)$$

Chary et al; (2006) performed extensive laboratory testing on sandstone samples collected from coal mining sites of Nevveli Lignite Corporation Limited (NLCL) and Singareni Collieries Company Limited (SCCL) and found following correlations between UCS and V_P:

For NLCL sandstone samples:

 $UCS = 0.1564V_P - 692.41 ----- (25)$ For SCCL sandstone samples: $UCS = 0.0144V_P - 24.856 ----- (26)$

Sharma and Singh (2008) carried out tests on 48 rock samples comprising of all three generic rock types to develop empirical mathematical relation between UCS and V_p described as follows:

UCS = $0.0642 V_p - 117.99 - (27)$

Yagiz (2011) proposed the following empirical equations between UCS and V_p after determining geotechnical properties of nine different rocks consisting of travertine, limestone and schist in the laboratory:

Linear relationship:

 $UCS = 49.4 V_p - 167 ----- (28)$ Power relationship:

 $UCS = 0.258 \ V_p^{\ 3.543} \ \text{-----} \ (29)$ The investigations of Kurtulus et al; (2011) on serpentinized ultrabasic cored rock samples for uniaxial compressive strength and ultrasonic pulse velocity (UPV) along and across foliations resulted in the following correlations between UCS and UPV:

For tests across foliation:

UCS = 0.0675 (UPV) - 245.13 ----- (30) For tests along foliation:

UCS = 0.0188 (UPV) - 71.04 ----- (31)

Hakan and Derya (2012) carried out laboratory experiments on carbonate rocks including onyx, travertine and limestone in order to develop correlation between UCS and index properties. They employed statistical regression analysis techniques to establish possible relationship between the UCS, ultrasonic wave velocity (V_p) and apparent porosity (n) for the three rock types as follows:

For onyx rock samples:

UCS= 14 ln $(V_p/n) - 96$ ----- (32) For travertine rock samples: UCS= 15 ln $(V_p/n) - 73$ ----- (33)

For limestone rock samples:

UCS= $0.0009 (V_p/n) + 38 ----- (34)$

MATERIALS AND METHODS

Sampling Source: The core samples of dolerite rock were collected as a result of exploratory drilling by the Geological Survey of Pakistan near the boundary of Jhang and Sargodha districts in Punjab province of Pakistan. The drill hole site was located 1.5 kilometers north of Chak 142 and 12 kilometers southwest of Sillanwali area, at a Latitude of 31°45'30" and a Longitude of $72^{\circ}29'50''$. The hole was drilled to a depth of 550 meters, divided into upper non-coring alluvium cover of 192.6 meters and below that dolerite coring run of 357.4 meters.

The GSP logged the cores and handed over the samples to the rock mechanics laboratory for further analysis. Six core boxes with a total length of 120 meters were received. Majority of the samples were damaged and were discarded. After sorting and visual examination the flawless samples were cut, trimmed and lapped to

standard sizes. Preparation of core samples and testing was conducted in accordance to standard methods adopted by ISRM and ASTM.

Laboratory Testing Procedure: A total of twenty three (23) sets of rock core samples were tested for direct UCS, point load index (I_{S(50)}) and sonic velocity (V_p). Direct UCS test was executed on rock cores with approximate length to diameter ratio 2.5:1 (Figure 1). The cores were pressed to fracture, at a very slow rate (to avoid dynamic loading) using a 200 ton Universal Testing Machine. Point load test was performed by loading cores on two pointed platens until failure occured (Figure 2). Core diameters with 54 mm, 42 mm and 30 mm were used to perform point load test. Corrections were applied for diameters other than 54 mm to calculate equivalent diameter and Is(50) was computed. The sonic velocity was measured by means of Portable Ultrasonic Nondestructive Index Tester (PUNDIT). The time taken by sound waves to pass through rock cores was recorded, and sound velocities were computed.



Figure 1 – Determination of uniaxial compressive strength of dolerite by direct testing method



Figure 2 – Determination of point load index of dolerite

RESULTS AND DISCUSSION

The test results of experiments performed for uniaxial compressive strength, point load index and sonic velocity on dolerite samples revealed that UCS of dolerite varied from 31.27 to 388.74 MPa with an average value of 189.61 MPa. The average values of point load index and sonic velocity were found to be 0.91 MPa and 7.33 km/s. The point load test values and sonic velocities computed ranged from 0.11 to 2.63 MPa and 6.62 to 7.85 km/s respectively.

The results were statistically scrutinized by using regression analysis and six statistical significant equations were found.

Correlations between UCS and I_{s} (50): The various relationships between UCS and Point Load Index (PLI) and their correlation coefficients were found through statistical regression analysis. All the three functions (linear, exponential and power) showed increase in UCS with increase in PLI values, thus representing a positive correlation between the two parameters. Figures 3, 4 and 5 present the results for linear exponential and power function, respectively. Among the results power function was the more reliable with greater coefficient of correlation thus it provided a better estimation of UCS for a wide range of $I_{s(50)}$ values. It can be seen that the power function was the most suitable with the data obtained. Although linear-fit curve was not an ideal one but it provided better prediction of UCS for lower values of Is(50).



Figure 3 - Linear relationship between UCS and I_{s50}



Figure 4 - Exponential relationship between UCS and Is50

Correlations between UCS and Vp: Various correlation relationships (linear, exponential and power) between Uniaxial Compressive Strength (UCS) and Sonic Wave Velocity (V_p) are presented in Figures 6, 7 and 8, respectively. All three relationships depict positive correlations between UCS and V_p . In this case, linear function exhibited statistically more realistic correlation with correlation coefficient of 79%.



Figure 5 - Relationship between UCS and $I_{\rm s50}$ based on power function

The relationship was not applicable for smaller values of V_p as the equation had a negative intercept of – 1413. The range of V_p for prediction of UCS may be 7.40 km/s to 8.00 km/s or above. It can be seen that exponential function gave a better estimation at lower values of V_p but it overestimated UCS at higher values. It can be observed from Figure 6 that power function was best-fit curve for the correlation between UCS and V_p . Although its correlation coefficient was slightly less than that of linear function but it better estimated UCS values over a wider range of V_p values.

Significance of derived equations: The statistical analysis of test results produced the following six prediction equations for UCS versus I_{s} (50) and UCS versus V_{p} :

UCS = 110.1Is + 89.87 (R = 71%) --- (35) $UCS = 85.52e^{0.718Is} (R = 67\%) ----- (36)$ $UCS = 202.71 Is^{0.633} (R = 80\%)$ ------ (37) UCS = 218.8Vp - 1413 (R = 79%) --- (38) $UCS = 0.003e^{1.455Vp} (R = 76\%) ----- (39)$ $UCS = V_p^{10.6} \times 10^{-7} (R = 76\%) ----- (40)$ 400.00 R = 79%320.00 240.00 UCS, MPa 160.00 80.00 0.00 6.50 6.75 7.00 7.25 7 50 7.75 8.00 Sonic Velocity, Vp (Km/s)



The statistical significance of all the six correlations was determined by the standard test wherein the computed t value $[t = r * (n-2)^{1/2} / (1-r^2)]$ was checked against a critical t value. If the t value from computation was more than the critical t value the correlation coefficient was considered to be statistically lesser or

where:

greater than zero, which means the relationship can be used for the prediction of the dependent variable from the independent variable. All the six relationships were found to have statistical significance.



Figure 7 - Exponential relationship between UCS and V_p



Figure 8 - Relationship between UCS and V_p based on power function

To check the estimation accuracy of these equations, the concept of "Confidence Interval" (CI) was used. For a normal distribution, the 95% confidence interval of mean is expressed as:

 $CI_{95\%} = 1.96 \text{ SD/SQRT} (n) ------ (41)$

SD = Standard deviation

n = No. of observation

The standard deviation, mean and 95% confidence interval values of the uniaxial compressive strength are given in Table 1, whereas Table 2 and Table 3 shows estimated values of UCS computed from derived prediction equations against measured UCS values. It can be observed that 95% of the predicted UCS values from UCS versus $I_{s\ (50)}$ equations fall within 95% CI range. Whereas all the predicted UCS values from UCS versus V_p equations fall within 95% CI range.

Table 1 - Mean UCS and Standard Deviation of tested rocks

Mean UCS	Standard Deviation	UCS Range
T*	SD	T* <u>+</u> 1.96 (SD)
(MPa)	(MPa)	(MPa)
189.61	89.55	14.10 to 365.13

Table 2 - Validation of predicted equations of UCS from Point Load Index values

Sr.	Sample title	UCS (MPa)	$I_{s\ (50)}$	Estimated values		
No.				UCS = 110.1Is + 89.87	$UCS = 85.52e^{0.718Is}$	$UCS = 202.71 Is^{0.633}$
1	S-1	331.38	2.63	379.43	565.15	373.86
2	S-2	46.12	0.20	111.89	98.73	73.19
3	S-3	233.71	0.71	168.04	142.38	163.20
4	S-6	71.23	0.48	142.72	120.71	127.38
5	S-8	31.27	0.11	101.98	92.55	50.13
6	S-10	214.26	1.63	269.33	275.64	276.18
7	S-11	140.00	1.01	201.07	176.61	203.99
8	S-12	250.45	1.16	217.59	196.69	222.68
9	S-13	179.49	1.25	227.50	209.82	233.46
10	S-14	156.71	0.40	133.91	113.97	113.50
11	S-19	245.49	0.90	188.96	163.20	189.63
12	S-20	312.21	1.10	210.98	188.40	215.32
13	S-21	388.74	1.58	263.83	265.92	270.79
14	S-22	258.09	1.16	217.59	196.69	222.68
15	S-23	83.15	0.31	124.00	106.84	96.58
16	S-24	126.70	0.18	109.69	97.32	68.46
17	S-27	225.91	0.51	146.02	123.34	132.36
18	S-28	139.81	0.90	188.96	163.20	189.63
19	S-29	199.77	0.86	184.56	158.58	184.25
20	S-31	234.08	0.49	143.82	121.58	129.05
21	S-32	154.63	1.06	206.58	183.06	210.33
22	S-34	191.40	1.36	239.61	227.06	246.27
23	S-35	146.52	0.84	182.35	156.31	181.53

Sr.	Sr. No. Sample title	UCS (MPa)	Vp (Km/s)	Estimated values		
No.				UCS = 218.8Vp - 1413	$UCS = 0.003e^{1.455Vp}$	$UCS = V_p^{10.6} \times 10^{-7}$
1	S-1	331.38	7.85	304.58	273.86	305.93
2	S-2	46.12	6.94	105.47	72.86	82.87
3	S-3	233.71	7.69	269.57	216.98	245.93
4	S-6	71.23	7.12	144.86	94.68	108.71
5	S-8	31.27	6.88	92.34	66.77	75.59
6	S-10	214.26	7.62	254.26	195.97	223.21
7	S-11	140.00	7.49	225.81	162.20	186.00
8	S-12	250.45	7.49	225.81	162.20	186.00
9	S-13	179.49	7.21	164.55	107.92	124.20
10	S-14	156.71	7.28	179.86	119.49	137.59
11	S-19	245.49	7.55	238.94	176.99	202.41
12	S-20	312.21	7.50	228.00	164.57	188.65
13	S-21	388.74	7.60	249.88	190.35	217.08
14	S-22	258.09	7.76	284.89	240.24	270.73
15	S-23	83.15	6.62	35.46	45.74	50.25
16	S-24	126.70	7.27	177.68	117.77	135.60
17	S-27	225.91	7.67	265.20	210.76	239.24
18	S-28	139.81	7.05	129.54	85.51	97.90
19	S-29	199.77	7.04	127.35	84.27	96.44
20	S-31	234.08	7.57	243.32	182.22	208.17
21	S-32	154.63	7.07	133.92	88.03	100.89
22	S-34	191.40	7.11	142.67	93.31	107.11
23	S-35	146.52	7.12	144.86	94.68	108.71

Table 3 - Validation of predicted equations of UCS from P-wave velocity

Direct UCS prediction equations from indirect UCS tests have been developed. The empirical relations so developed in this study were also found to be statistically significant. Among these relationships, power functions both for point load test and p-wave velocity can be employed reliably for prediction of UCS. It was found that linear and exponential relationships could also predict UCS for the same rock type but were less reliable. The results presented are appreciable for the rock type studied.

Conclusions: A critical review of literature revealed that there had been significant work done in establishing statistical significant relationships. In the present work it was established that dolerite rock provided an indirect, fast assessment of one of the important rock strength parameters.

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