

# Fluid Substitution Analysis Using Gassmann's Equation Modification on Carbonate Environment

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**Abstract**—Gassmann's equation can be used to determine the velocity of compressional waves that pass through rocks with various pore fluid contents, using fluid substitution concept, but is generally applied to certain conditions only (physical rock properties). Carbonate rock has properties in contrary of Gassmann's assumption; is a heterogenic, anisotropic rock and does not have a well-interconnected pores. In this research, secondary data from laboratory measurements are used, consisting of carbonate rocks (limestone and dolomite) to test modified Gassmann's equation on carbonate rocks. Two approaches of Gassmann's equation are used by using k-dry and k-1 components, which are the value of compressional modulus bulk of saturated rocks. The result of both approaches shows that the usage of k-1 component is more optimal to be applied to carbonate rocks because it does not use k-dry component, which should only be used on field measurements, as there is a difference in the environment condition (air, temperature, and pressure) on reservoir and laboratory.

**Index Terms**— Bulk modulus, Carbonate, Compressional wave velocity, Gassmann, Porosity.

## I. INTRODUCTION

CARBONATE rock is a hydrocarbon reservoir rock with a complex structure. During deposition, porosity is generated between the particles in the rock in the same way for both reservoir types; sandstone and carbonate. After precipitation, diagenesis plays an important role in the evolution of porosity, where carbonates undergo great compaction, dissolution, precipitation and cementation; modifying the mineralogy and geometry of pore spaces repeatedly. As a result, the carbonate reservoir is very heterogeneous and has a complex microstructure [1]. Due to its complex nature, predicting the reservoir character through its seismic properties is quite difficult.

The Gassmann equation (1951) is the most popular approach for determining the velocity at various pore fluid contents. This equation allows the calculation of the wave velocity for any pore fluid if one of the velocity is known (measured) in one type pore fluid, which is the concept of "fluid substitution". The Gassmann equation relates the elastic modulus of a

saturated porous material with a dry (empty) matrix and the compressibility of the fluid. This equation is also widely used in modelling the acoustic properties of fluid saturated rocks, especially sand, sandstones and carbonates. Because this equation has assumptions and limitations, such as constant shear modulus and interconnected pore spaces, this does not apply to carbonate environmental rocks. The assumptions and limitations of the Gassmann equation are macroscopically homogeneous and isotropic rocks. Second, in the interconnected pore, there is an equilibrium fluid pressure and there is no trend of pore pressure due to the passing waves, so that the low frequency allows the balance of pore pressure in the pore space. Therefore, the Gassmann equation works best for seismic frequencies (<100 Hz) and high permeability [2]. Third, the rock-fluid system is closed (not drained), that is, no fluid can flow into or out of the volume under consideration during the passage of the wave. Fourth, the pore fluid does not interact with the solid material or rock frame. Finally, the passing waves produce movement (displacement) of the entire rock section, but no relative motion between the solid and fluid rock frames [3]. In this study, the aim is to perform a test by developing the Gassmann equation for calculating the velocity of carbonate rocks. Therefore, this approach can assist in the understanding of elastic properties carried out on a laboratory scale.

## II. LITERATURE STUDIES

Porosity states the ratio of pore volume inside a rock with its entire volume. The value of rock's porosity affects the ability of a rock to store fluid inside it, and is defined as [4]. Porosity can be stated on the following equation:

$$\phi = \frac{\text{Pore Volume}}{\text{Rock Volume}} \times 100\% \quad (1)$$

Where  $\phi$  is porosity pore volume is the volume of pore of a rock, and rock volume is the total volume of rock sample.

### A. Physical Rock Properties

Density is one of rock's physical property that affects other properties of rock. Density is defined as the ratio of rock's mass

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(m) to its volume (v), and can be written as the following equation,

$$\rho = \frac{m}{v} \quad (2)$$

Where  $\rho$  is rock's density, m is rock's mass, and v is rock's volume. In international standards, the unit of is kg / m<sup>3</sup>. Because rocks are heterogeneous, special density definitions are needed related to the constituent material components that construct a rock

### B. Rock Elasticity Properties

Rock elasticity shows the relationship between stresses of a rock that causes it to deform. A material is said to have elastic properties if it stretches when given stress, and returns to its original state when the stress is removed. Rocks have elasticity parameters such as Poisson's Ratio, Young's Modulus, Bulk Modulus, Shear Modulus, and Lamé's constant. These parameters are known as the Elasticity Constants. Bulk modulus (k) is also known as rigidity modulus or incompressibility. Bulk modulus is the ratio of axial pressure to volume deformation. The following is an equation for the bulk modulus.

$$K = \frac{\Delta P}{\Delta v/v} \quad (3)$$

Where K is bulk modulus,  $\Delta P$  is axial pressure, and  $\Delta v/v$  is volume deformation. The shear modulus is a ratio between the components of shear stress and shear strain. It can also be stated as the stiffness of the rock to be shifted. Here is the equation of the shear modulus,

$$G = \frac{\sigma_{i,j}}{2s_{i,j}} = \frac{F/A}{\Delta x/l} \quad (4)$$

where

G is a shear modulus, s is a component of stress shear, is a component of strain shear. There are two waves that propagate below the surface; P wave and S wave. The P wave travels at the speed of  $V_p$  while the S wave travels at the speed of  $V_s$ . Below are the equations of P wave and S wave velocity,

$$V_p = \sqrt{\frac{(\lambda+2\mu)}{\rho}} \quad (5)$$

$$V_s = \sqrt{\frac{\mu}{\rho}} \quad (6)$$

### C. Gassmann's Assumption

The Gassmann equation (1951) is used to determine the velocity of a rock containing various fluids in its pores (pore fluid contents), if the velocity is known (measured) in one pore fluid. This is the concept of fluid substitution. The Gassmann equation relates the elastic modulus of a saturated porous material with a dry (empty) matrix and the compressibility of the fluid. The Gassmann equation can also be used to provide a simple model that can estimate the effect of fluid saturation on the bulk modulus, and can be used to calculate fluid

substitution. Gassmann equation can be written as follows,

$$K_s = K_d + \Delta K_d \quad (7)$$

$$\Delta K_d = \frac{K_0 (1 - \frac{K_d}{K_0})^2}{1 - \phi - \frac{K_d}{K_0} + \phi \times K_0 / K_f} \quad (8)$$

The equations above state that the fluid in rock pores affects the bulk modulus, not the shear modulus. The Gassmann equation has four assumptions, including:

- i. Isotropic, elastic, monomineral, and homogeneous porous material.
- ii. The pore space is well-connected and in a state of equilibrium pressure (zero-frequency limit).
- iii. Closed medium without movement of pore fluids across boundaries.
- iv. There is no chemical interaction between fluid and rock (constant shear modulus)

## III. METHODOLOGY

### A. Rock Samples

This research uses two types of carbonate rock samples. First, rock samples are originated from the Southern part of France, near Cassis and La ciotat. 33 dry limestone are used with their own mixtures; limestone (grain stone), limestone (wacke-pack stone), quartz-rich limestone (grain stone), quartz-rich limestone (wacke-pack stone), and argillaceous quartz-rich limestone. Second, rocks samples are originated from Great Oolite Limestone formation from Southern England, rocks from hydrocarbon reservoir. This formation consists of predominantly oolitic, skeletal, and oncolytic grain stone, also pack stone. 41 samples are used, with several facies variations from oolitic and skeletal grain stone until wacke stones. The rocks consist of calcite and dominant dolomite with less amount of quartz and feldspar minerals.

### B. Laboratory Measurements

Plugs with a diameter of 2.5 cm and length of 2 – 2.2 cm are used in laboratory measurements. Grain density is measured using helium-pycnometer on dry rock samples (on 105 Celcius degrees heat) and effective porosity is measured from bulk density and the derivative of grain density. Wave velocity is measured by using ultrasonic waves that are measured on dry rock samples brine-saturated (1000 pm of NaCl). Samples are placed in between the transmitter and receiver with a contact agent and axial pressure of 2 bars, in order to obtain better coupling between the samples and transducer. 80 kHz pulse is given to the transducer until mechanical pulse moves through the rock samples in a form of wave. The incoming waves are visualized on a computer screen with an oscilloscope storage. All measurements are dead-time corrected. The signal of the measurement has similar characteristics with seismogram.

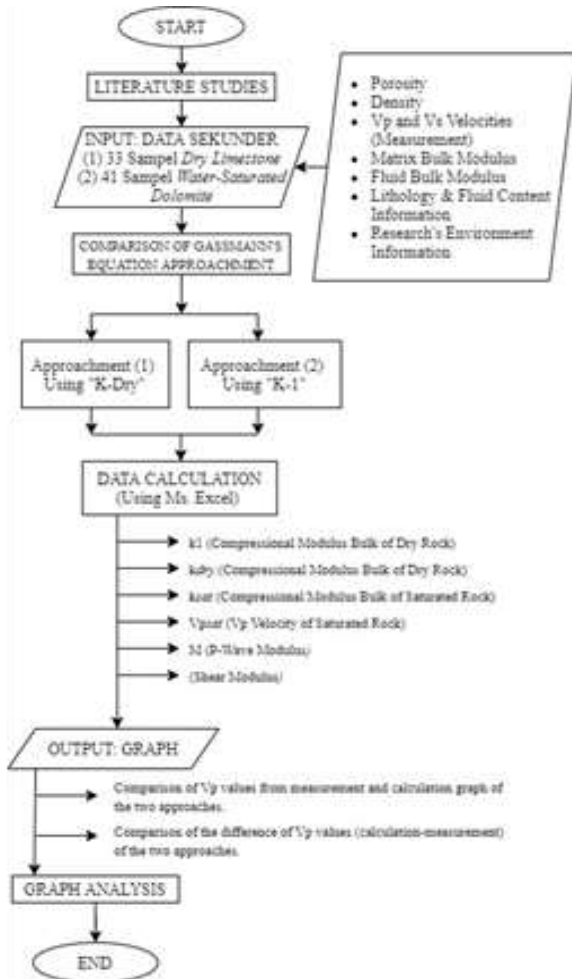


Fig. 1. Research Flowchart.

C. Research Methodology

This research is done to prove the usage of Gassmann's equation of carbonate rocks, which have different and contrary characteristics from Gassmann's ideal assumption, as stated on Part 2.4. Two approachments are used of Gassmann's equation to evaluate whether it can be applied on carbonate rocks or not. Both approachments of Gassmann's equation are done on rock samples to obtain the value of compressional wave velocity (Vp) on saturated rocks, which later will be compared with the measurement values. Hereby are the two Gassmann's equation approachments that are used in this equation.

$$k_{sat-1} = k_1 + \frac{(1 - \frac{k_1}{k_s})^2}{\frac{\phi}{k_{fl}} + \frac{(1-\phi)}{k_s} \frac{k_1}{k_s}}$$

$$V_{psat-1} = \sqrt{\frac{k_{sat-1} + \frac{4}{3}\mu}{\rho}}$$

D. Second Approach

Approach 2 modifies the Gassmann's equation, by only using the bulk modulus of dry rock frame (k-1 component) and not using k-dry component.

IV. RESULTS AND ANALYSIS

The Gassmann equation can be used to determine the velocity of waves passing through rocks with various pore contents, using the concept of fluid substitution. This equation can be applied to several ideal conditions, which is called the Gassmann assumptions. Carbonate rocks have characteristics that contradict Gassmann's assumptions. Carbonate rocks are anisotropic heterogeneous because they have the dominance of various content in rocks. Meanwhile, Gassmann's assumptions apply to isotropic heterogeneous rocks. Carbonate rocks do not have well interconnected pores. Meanwhile, Gassmann's assumption applies to rocks with well-interconnected pores. Carbonate rocks have a low shear modulus value for saturated rocks. Meanwhile, Gassmann's assumption states that rock fluid content does not affect the shear modulus value.

In this study, two literatures that have different rock sample properties were used. The first literature was taken from research locations in Southern France with samples of dry limestone carbonate rocks, or rock pores filled with air. In the research, the author uses two approaches, the first is the use of the k-dry component and the second is the use of the k-1 component, without the k-dry element. The following is a graph that shows the comparison of the Vp value between the measurement results and the calculation results.

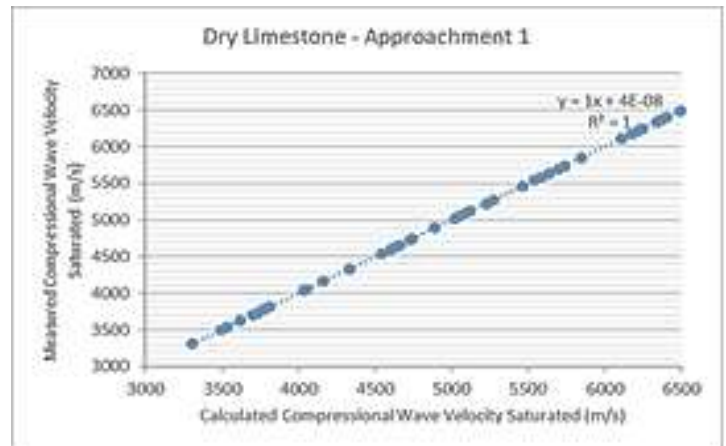


Fig. 2. Comparison Chart of Vp Measurements and Vp Calculation on Dry Limestone Using Approachment 1.

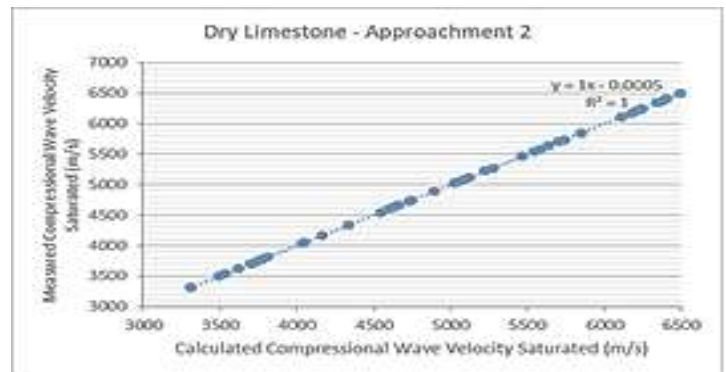


Fig. 3. Comparison Chart of Vp Measurements and Vp Calculation on Dry Limestone Using Approachment 1.

In the first approach, a graph of the comparison of the Vp value of measurement and calculation shows the same result, which is indicated by the regression equation "y = x". Approach one uses a k-dry component that can only be used during field measurements. In field measurements, there are factors of temperature and pressure from the reservoir, which affect the pore fluid content and incompressibility of rock samples. Thus, it is as if there is not the slightest change in the Vp value of measurement and calculation (which means that the equation cannot be applied optimally). Meanwhile, this research was carried out in laboratories that have different air conditions and environmental factors. Meanwhile, in the second approach, the graph of the comparison of the Vp value of measurement and calculation shows different results (there is a difference in the value between the measurement and calculation results). The second approach does not use the k-dry component in its calculations because it adapts to research activities carried out on a laboratory scale. This makes the second approach (using the k-1 component only) more sensitive to carbonate rock samples than the first approach (using the k-dry component). The second approach shows that there is a difference between measurement and calculation, although it is very minor (see Fig. 2 and Fig. 3).

Based on the graph above, a comparison of the relationship between porosity and the difference between Vp Calculation and Vp measurements has been obtained in water-saturated dolomite, using approaches 1 and 2 as in the above discussion. In the two graphs above, the porosity is plotted on the Y-axis, while the difference between Vp calculation and Vp measurement is plotted on the X-axis. It can be seen that the graph in Figure 4.8 provides better data distribution results than the graph in Figure 4.7. In approach 1, because the values of Vp measurement and Vp calculation are the same, it will result in a difference value = 0. This causes the point distribution to only be on the y-axis (porosity). In contrast to the results of the graph using the second approach, which is relatively spread out with the dominant data being at a difference of 3-4. This shows that the second approach where the Gassmann equation uses k-1 components gives better results than the first approach where the Gassmann equation uses k-dry. The first approach is considered not giving maximum results because the k-dry component does not represent the sample conditions in the field and in the laboratory. This is because the environmental conditions in the reservoir have a different level of compressibility from the air in the laboratory.

Based on the two graphs above, the relationship between porosity and P wave velocity is inversely proportional. When the porosity value is high, the P wave velocity is low. The greater the porosity in the rock, which indicates the number of pores in the rock that can be filled with fluid. This slows down the rate of P waves passing through the rock. In contrast, when the porosity value is low, the P wave velocity is high. The smaller the porosity in the rock, the denser and denser the rock.

This accelerates the rate of P waves passing through the rock (see Fig. 4 and Fig. 5).

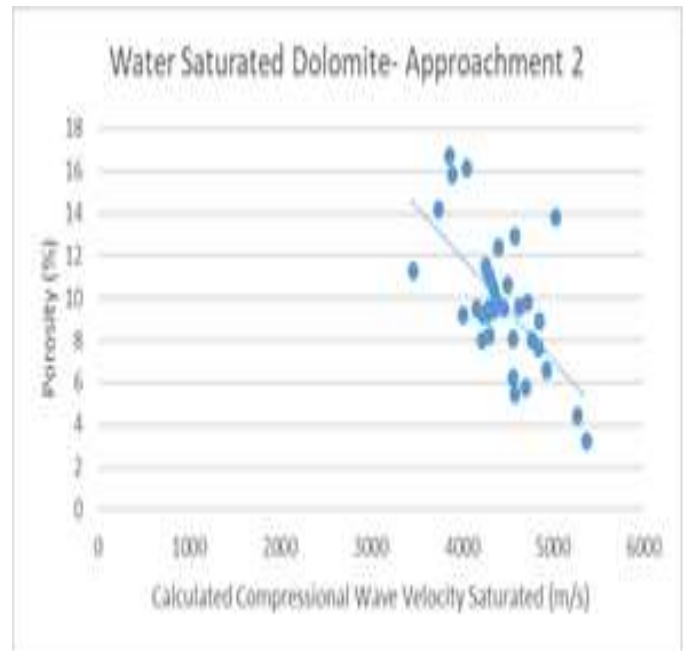


Fig. 4. Relationship between Porosity and P Wave Velocity in Dry Limestone Rocks

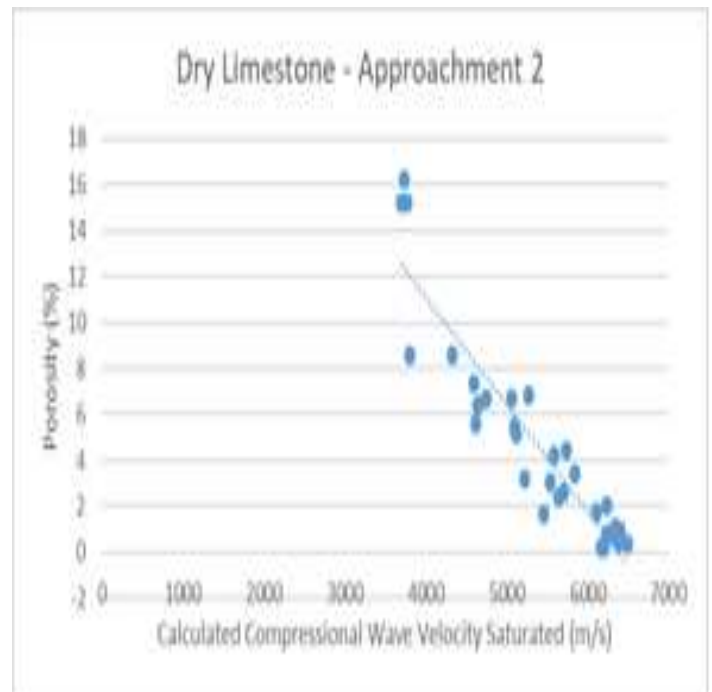


Fig. 5. Relationship between Porosity and P Wave Velocity in Water-Saturated Dolomite Rocks.

## V. CONCLUSION

In this paper, the application of the modified Gassmann formula is used to find value of saturated P wave velocity (Vp). Carbonate rocks (limestone and dolomite) is used for this study,

so it can be analyzed the Gassmann equation, which is generally used for isotropic homogeneous that have interconnected pores, can be used in anisotropic heterogeneous that have poor interconnected pores (carbonate). As we can see from the comparison graph between  $V_p$  calculated has the same result with  $V_p$  measurement in the dry limestone, this is presumably due to the absence of fluid that affects the value of  $V_p$  saturated using the Gassmann equation's approachment 1 and approachment 2. And in the same time, from the comparison graph between  $V_p$  calculated has a result that is relatively similar with  $V_p$  measurement in the water-saturated dolomite, the value of  $V_p$  calculated is higher than  $V_p$  measurement. This analysis can be validated with the graph of difference  $V_p$  calculation and  $V_p$  measurement. It showed the difference is non-existent in dry limestone samples and there is few difference in water-saturated dolomite. And then, from the graph of the porosity and  $V_p$  calculation is suitable with the theory, the higher  $V_p$  that means slower P wave propagates, due to the smaller porosity of the rocks. Further research on application of modified Gassmann equation can be used to determine the distribution of fluids between gas and water in hydrocarbon exploration and monitoring, and can be used for seismic interpretation and modelling in carbonate environments under critical hydrogeological conditions.

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