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OPTIMAL DESIGN OF 16 BAR TRUSS STRUCTURE BY PATTERN SEARCH METHODS

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ABSTRACT: The focus of this research was to formulate optimization model of 16-bar trusses along with stress, stability and deflection constraints. The derivative free methods were used for the optimization of engineering design problems. These methods were basically designed for unconstrained optimization problems. In formulated optimization truss problems the constraints were handled by using exterior penalty functions. The results of the truss optimization model were obtained by using MATLAB which demonstrated the effectiveness and applicability of these derivative free methods. It was concluded that the results of Nelder-Mead method were not acceptable due to their far away convergence even its number of function evaluations were smaller than number of function evaluations of Multi-Directional Search method and Hooke and Jeeves method. By comparing the optimal function values obtained by these three methods, the performance of Hooke-Jeeves method was better than the other two methods.

Keywords: derivative free methods, penalty function, structural optimization, truss structure, unconstrained optimization.

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INTRODUCTION

The optimization problems appear in nearly all ranges of life like assembling, scheduling, engineering and business. Utilizing optimization procedures, the best results of the problems are obtained by using limited measure of restricted assets (Rao, 2009).

Two principle procedures of optimization, specifically, derivative Based Methods (DBM's) and derivative Free Methods (DFM's) are, no doubt utilized frequently (Tabassum *et al*, 2015). Among the direct search methods the concentration is on Hooke -Jeeves (HJ) method (Hooke and Jeeves, 1961), Nelder-Mead (NM) method (Price *et al.*, 2002) and Multi-Directional Search (MDS) method (Torczon, 1989). These methods are used for unconstrained optimization problems. These DFM's connected to constrained optimization problems by changing them into unconstrained optimization problems by using the penalty function (Conn *et al.*, 2009).

In the early years when the derivatives of functions were challenging to calculate, the direct search methods were popular, but recently, numerous tools for robust and automatic differentiation are available as well as modeling languages that compute derivatives automatically (Price *et al.*, 2002). In spite of all this, direct search methods have their own importance. Particularly the maturation of simulation-based optimization has made it difficult to use derivative based methods. Moreover, the objective functions which are not numeric in nature cannot be optimized by derivative based methods. In addition, the objective functions which are not numeric in nature cannot be simplified by derivative based strategies (Andrad, 1998).

For calculating different sorts of optimization problems lot of direct search methods have been produced by the analysts. A definite investigation of these systems, with recorded foundation, might be found (Lewis *et al.*, 2000).

The thought of this system is to change the constrained optimization problems to an unconstrained one by adding or subtracting the values from the objective function focused around constraint violation present in the result (Ashok and Chandrugupta, 2011).

This paper is based on several operations which have been developed for DFM's. Secondly it presents constrained handling techniques for optimization problems which have occurred in DFM's and also handles as to how the model behaves with specific constrained. Finally the performance of the methods are compared.

MATERIALS AND METHODS

The motivation for this research was to modify engineering truss problems. The derivative free methods were used for the optimization of engineering design problems. These methods were basically designed for unconstrained optimization problems. In formulated optimization truss problems the constraints were handled by using exterior penalty functions (Tabassum *et al*, 2015).

The structural optimization problems might be formally detailed as minimizing the objective functions, subject to demand on mechanical demonstrations (Isaac and Ohsaki, 2010). The aggregate structural volume (or weight) was typically allocated as the target capacity, in the light of the fact that it was a basic prerequisite to decrease the weight of the aviation and mechanical structures. Structural optimization may be subdivided into shape optimization and topology optimization (William, 2001). Structural optimization problems could be attractively easy to figure, while might be composed as, find x to minimize subject to $g(x) \leq 0$. Here f was the objective function and g was the constraints. Such problems are called numerical programming problems

$$\text{Min } f(x) \qquad \text{Subject to } g(x) \leq 0$$

Derivative free methods analyzed the tools to create structural optimization that was capable of size and shape optimization of truss and frame structures. Usability was increased by including graphical viewing utilities for structure visualization and optimization progress. The objective of the structural optimization was the minimization of volume and weight with optional stress and displacement soft-constraints. These problems deal with mixed continuous and discrete search spaces, which can create non-smooth and deceptive fitness landscapes. The optimization model conducted to show the validity of the derivative free methods and the feasibility of use on real engineering problems (Brian, 2005).

Development of N bar truss model: Consider N bar trusses, in these trusses it was attempted to minimize the weight under stress constraint. The design variables were the cross sectional area.

Objective function: These kind of problems have considered the weight of the general truss as the objective function. The parameters ρ and L_i were the material thickness and length of i^{th} part, separately.

$$f(A) = \sum_{i}^{n} \rho A_i L_i$$

$$A_i \geq 0, \qquad i = 1, \dots, n$$

Constraints: Firstly, it points out the area and the amount of fundamental nodes for supports and loads. Accordingly, a feasible truss must have all the fundamental nodes.

Secondly, the truss must not deflect more than the allowable limit due to the application of loads as is shown below.

$$G_2 = \sum \delta_k^{max} - \sum \delta_k(A) \quad , k = 1, 2, \dots, n$$

Thirdly, the trusses of different topologies were created on the fly, some of them may be statically determinate and some of them may be statically uncertain. Hence, we have utilized derivative free strategies to compute the stress and deflection as is presented under.

$$G_3 = \sum S_j - \sum \sigma_j(A) \geq 0 \quad , \quad j = 1, 2, \dots, m$$

Finally, in a feasible truss all members must have stress within the allowable strength of the material. Some bar trusses had compressive force and these became compressive stress constraint and some had tensile force and these became tensile stress constraint as given below.

$$G_4 = \sum T_j - \sum \sigma_j(A) \geq 0 \quad , \quad j = 1,2,\dots,m$$

$$G_5 = \sum C_j - \sum \sigma_j(A) \geq 0 \quad , \quad j = 1,2,\dots,m$$

In the above non-linear programing problems where

ρ = density of the material (focused that this specific objective function did not depend on any state variable)
S_j = allowable strength of the material,
T_j = allowable tensile of the material,
δ_k^{max} = allowable deflection in the truss and
C_j = allowable compressive strength of the material.

It was recommended that the cross sectional areas must be non-negative

$$A_i \geq 0, i = 1, \dots, n.$$

Hooke-Jeeves Method: For an N-dimensional problem HJ method was studies, which required an initial point x_0, a set of N linearly independent search directions v_i, step-length parameters $\delta_i > 0$ and a parameter $\mu > 1$. Method used two types of moves given below:

Exploratory Move: This move was made on the current point by investigating along each direction according as following formula:

$$x_{new} = x_0 \pm \delta_i v_i \quad \text{for all } i = 1, 2, 3, \dots, N.$$

Pattern Move: When exploratory move completed and accomplished successfully then pattern move was executed, by jumping from present base point along with a direction connecting and a new point was found. Once a pattern move was established it was possible to move as much as allowed. An enlargement parameter ζ, $\zeta \geq 1$, is used for this purpose. The pattern direction is found by the formula $d = z_E - z_b$. Therefore the new point, through pattern move, is found by

$$y_b = z_E + \zeta d = z_E + \zeta(z_E - z_b).$$

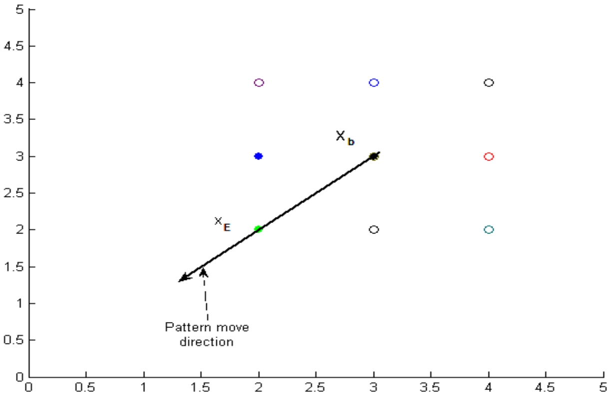

Fig-1: Pattern move direction

Nelder-Mead Simplex Method: While Considering the Nelder-Mead Simplex method the initial simplex with three initial points y^0 = *Best Point*, y^1 = *Good Point*, y^2 = *Worst Point* were taking the centroid y^C of best and good points. Reflected the worst point through centroid, the y^r became the new point, which having equidistance from y^C to y^2. In this method there were several operations to be performed. Reflection occur when $y^1 \geq y^r > y^0$.

Mathematically, the reflected point y^r was given by

$$y^r = y^C + \delta^R(y^c - y^n)$$

Expansion occur when $y^1 \geq y^0 > y^e$.

Mathematically, the expanded point y^e was given by

$$y^e = y^C + \delta^e(y^c - y^n)$$

In contraction when reflection point lies between the good and best vertex and it was generated two types. Outside contraction occur when $y^2 \geq y^r > y^1$.

Mathematically, the expanded point y^{OC} was given by

$$y^{OC} = y^C + \delta^{OC}(y^c - y^n)$$

Inside contraction occur when $y^r \geq y^2$. Mathematically, the expanded point y^{iC} was given by $y^{IC} = y^C + \delta^{iC}(y^c - y^n)$. If no one from the above conditions was satisfied then shrink was produced.

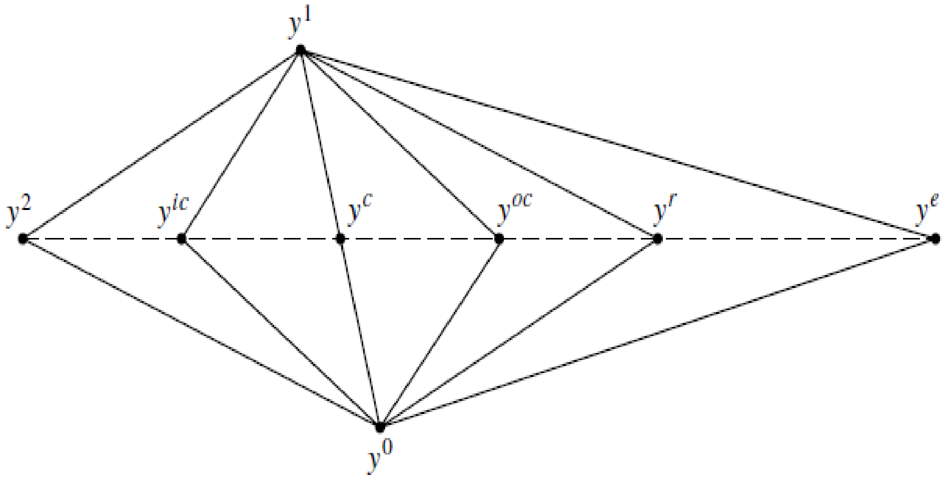

Fig-2: Steps of Nelder–Mead method.

Multi-directional Search method: In N- dimensional problem method started with a simplex of N+1 points. The method generated N points along N linearly independent search directions. The method used the following operations:

Reflection: The worst and good point was reflecting at the best point.

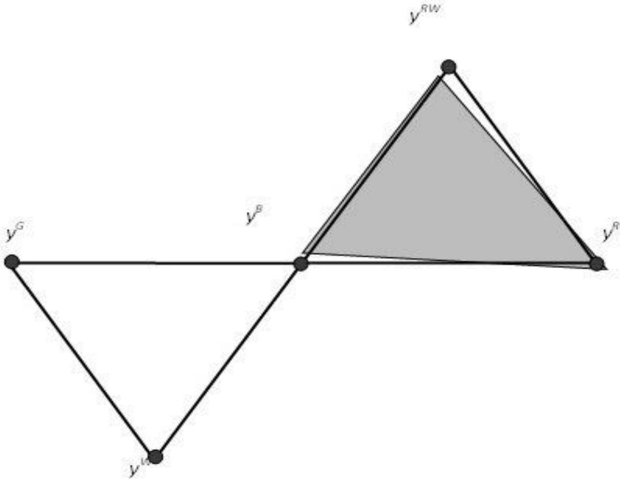

Fig-3: Reflection

Expansion: If the value of the reflection points was less than the best point then expansion was performed.

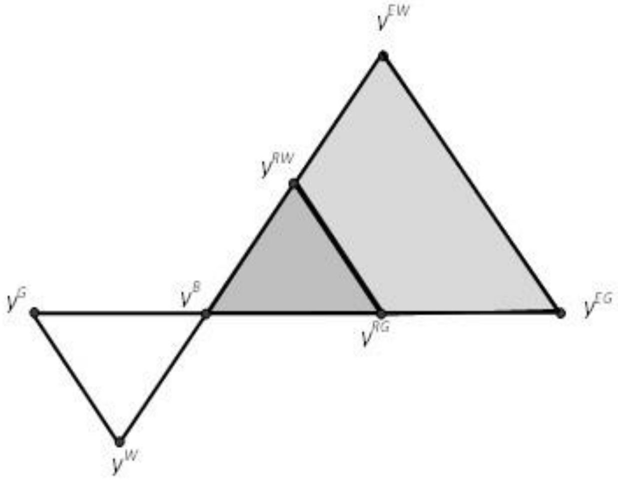

Fig-4: Expansion

Inner Contraction: If the values of the reflection points was not less than the best point then contraction was performed.

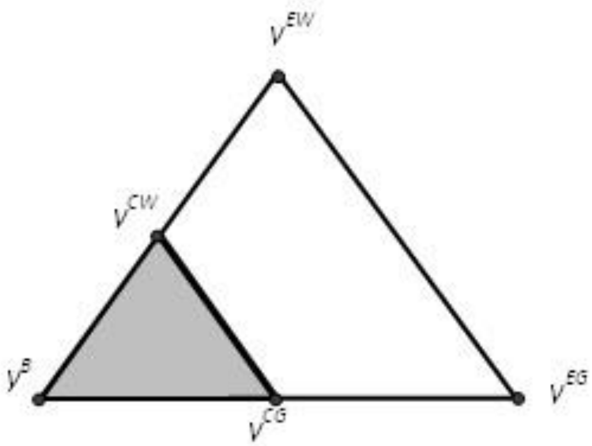

Fig-5: Contraction

Formulation of sixteen bar truss problem: While considering the sixteen bar truss problem. The bars *AB, BC, CD, DE, AF, FG, GH, HI and IJ* changed length from the other bars and young modulus E. It was to minimize the weight, the only constraint was a limit on the tip deflection *δ*. The design variables were the cross sectional areas $A_i, i = 1, \dots, 16$. The objective function or total volume of the truss became as under

$$minimize\ \sum_{i=1}^{16} L_i x_i \quad (1)$$

$$Subject\ to\ \delta_x \le \delta_u \quad (2)$$

$$\frac{F_i f_i L_i}{E x_i} \le \delta_u \qquad i = 1, \dots, 16 \quad (3)$$

The bound constraint $x_i \ge 0$ was enforced as $x_i \ge x_i^L$ where $x_i^L = 10^{-6}$. The above problem in (1) to (3) could also be stated as minimizing the weight of the truss structure subject to an upper limit on compliance (or lower limit on the stiffness). The expression for the tip deflection *δ* was given as under

$$\delta_u = \sum_{i=1}^{16} \frac{F_i f_i L_i}{E x_i}$$

Where *Fi* and *fi* were the forces in the element *i* due to the applied load *P* and due to a unit load at the point where *δ* was measured, respectively. Here, the tip deflection was evaluated at the point (and direction) where *P* was applied. Thus, *Fi* = *Pfi*. Moreover, *fi* was independent of x because the structure was statically determinate. The values of *fi*, obtained using force equilibrium at each node in the structure, are given below

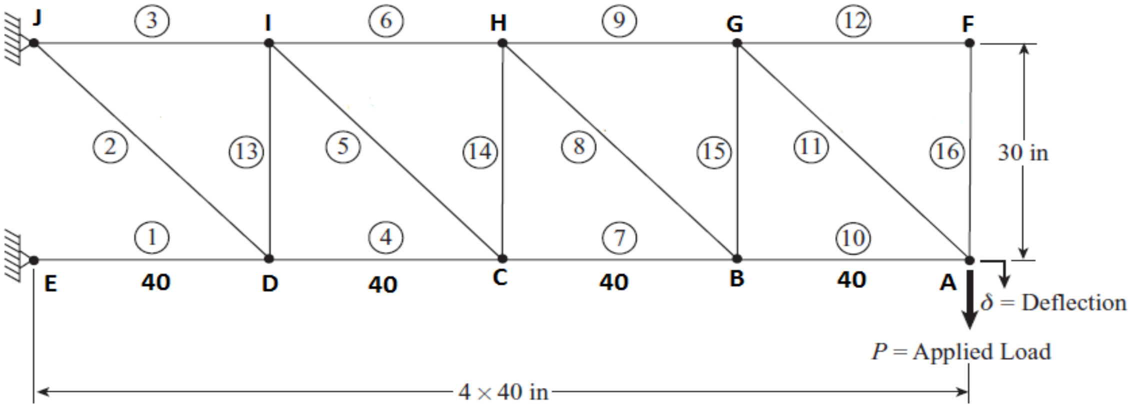

Fig-6: Sixteen bar truss

In fact, as $c_i = \frac{F_i f_i L_i}{E \delta_U}$ then the convex constraint in (*2*) can be written as

$$\sum_{i=1}^{16} \left(\frac{c_i}{x_i} - 1\right) \le 0$$

Table 1. Parameters table for sixteen bar truss.

Parameter	Description	Values
P	Applied load	25,000*ksi*
g	Specific gravity	.1lb/in^3
E	Modulus of Elasticity	30 * 10^6*psi*
x_i^L	Lower limit of cross-sectional area	0.1 *in*
x_i^U	Upper limit of cross-sectional area	35 *in*
A	Deflection parameter	1/16

The deflection constraint was

$$\delta = \frac{F_1 f_1 L_1}{E x_1} + \frac{F_2 f_2 L_2}{E x_2} + \frac{F_3 f_3 L_3}{E x_3} + \frac{F_4 f_4 L_4}{E x_4} + \frac{F_5 f_5 L_5}{E x_5} + \frac{F_6 f_6 L_6}{E x_6} + \frac{F_7 f_7 L_7}{E x_7} + \frac{F_8 f_8 L_8}{E x_8} + \frac{F_9 f_9 L_9}{E x_9}$$

$$+\frac{F_{10}f_{10}L_{10}}{Ex_{10}}+\frac{F_{11}f_{11}L_{11}}{Ex_{11}}+\frac{F_{12}f_{12}L_{12}}{Ex_{12}}+\frac{F_{13}f_{13}L_{13}}{Ex_{13}}+\frac{F_{14}f_{14}L_{14}}{Ex_{14}}+\frac{F_{15}f_{15}L_{15}}{Ex_{15}}+\frac{F_{16}f_{16}L_{16}}{Ex_{16}}$$

The objective function was

$$f(A)=L_1x_1+L_2x_2+L_3x_3+L_4x_4+L_5x_5+L_6x_6+L_7x_7+L_8x_8+L_9x_9$$
$$+L_{10}x_{10}+L_{11}x_{11}+L_{12}x_{12}+L_{13}x_{13}+L_{14}x_{14}+L_{15}x_{15}+L_{16}x_{16}$$

The above constrained optimization problem was converted into unconstrained one using exterior penalty function approach in the following form

$$\varphi(\mu)=\min_{x\geq x^L} imize\left\{\sum_{i=1}^{16}L_ix_i+\mu\left(\frac{c_i}{x_i}-\alpha\right)\right\}$$

RESULTS AND DISCUSSIONS

Direct search methods were popular because of their simplicity, flexibility, and reliability (Lewis *et al,* 2000). These methods have been shown to satisfy the first-order necessary conditions for a minimizer i.e., convergence to a stationary point (Lucidi and Sciandrone, 2002). It seemed remarkable that the given direct search methods neither required explicit derivative nor estimated derivative information. In most of the direct search methods a set of directions that span the search space was sufficient information to investigate the local behavior of the function (Rios and Sahinidis, 2012). To reduce the step length safely the set of directions had been queried (Nelder and Mead, 1965).

The stochastic combinatorial optimization approach based on Monte Carlo Method was used to solve 16 bar space trusses (Atusuhi and Hoshiya, 1996). The structural optimization problems with frequency constraints consisting 10 bar plane truss and some other spaces trusses were solved by using interior point trust region method (Zhenglei *et al*, 2013). Genetic-based hybrid algorithm that combines the exploration power of Genetic Algorithm (GA) with the exploitation capacity of a phenotypical probabilistic local search algorithm was presented efficiently on the optimal design of planar and space structures (Gholizadeh and Barati, *2012).* The Artificial Bee Colony algorithm with an adaptive penalty function approach was proposed to minimize the weight of different truss structures (Mustafa, 2010). The Ant Lion Optimizer was based on the hunting mechanism of Ant-lions in nature. The new algorithm was examined by designing three truss and frame design optimization problems (Talatahari, 2016). The fundamental concepts and ideas of mine blast algorithm were derived from the explosion of mine bombs in real world. The efficiency of the proposed optimizer was tested via the optimization of several truss structures with discrete variables (Ali *et al,* 2012).

In the above mentioned references the researchers presented the results of slightly different types of trusses design problems using various optimization techniques. The optimization problem of 16 bar truss and the methods to solve the problems presented in this paper were hardly available in the literature.

This paper presented a formulation and solution of new optimization model the so called 16 bar plane trusses model. The fundamental concept and idea to formulate this model was derived from the work represented by (Zhang *et al,* 2003 and Li *et al,* 2009). Computational results obtained from truss optimization problems clearly illustrated the attractiveness of the methods for handling problems with many design variables and constraints. In addition, fast convergence rate to reach the best solution and also low computational cost verified the potential for solving complex optimization problems.

Classical discrete structural optimization are presented here. These were intended to show the efficiency and accuracy of methods mentioned above. The sketch drawing of sixteen–bar truss is shown in figure 6. There were sixteen bar members and ten nodes. The two nodes at the left end (node E and node J) were pinned to prevent any displacement in both directions. The height of the truss structure was 30 in. Applied load of P=25000 *ksi* was applied on the node A, in the y direction. For this problem, the design variables were sixteen cross-sectional areas. The variables 1 through 16 represented the cross-sectional areas of members were 1 through 16, respectively. A lower limit of the cross-sectional area of 0.1in^2 and an upper limit of cross-sectional area of 35in^2 were enforced on each member. The objective function was the total material weight of the structure. The modulus of the material was 30*10^6 psi and the specific gravity was 0.1 lb/in^3. Displacement limit of ±2.0 in was imposed on all nodes in down word direction, and the limiting value of stress in each member was ±25,000 psi.

The structural optimization problems had been solved by (Ringertz, 1988) using generalized Lagrangean Method and Branch and Bound method as reported by (Adeli, 1991) using general Geometric Programming; by (Li, 2003) using Guide-weight Method; by (Tang and Gu, 2001) using Reproduction GA; by (Wu and Wang, 2002) using Parallel GA and other researchers also solved the problem.

The best results were acquired from sixteen bar truss by applying HJ method where the initial guess was taken in the range of 1 to 10. Many iterations were performed by taking different values in the above mentioned range and the function value in each iteration remained very close to *391.3281*. It means that this method showed the consistent performance for various initial guesses. The optimal point (4.722, 0.2930, 1.1875, 0.8906, 0.9297, 0.2969, 0.2969, 0.2930, 0.0625, 0.0625, 0.1172, 0.0508, 0.0508, 0.0508, 0.0508, 0.0508) was obtained after 896 function evaluations.

The best results were achieved from sixteen bar truss by applying NM method, where the initial guess was taken in the range of 1 to 10. Many iterations were performed by taking different values in the above mentioned range and the function value in each iteration remained very close to *602.9846*. It means that the method showed the consistent performance for various initial guesses. The optimal point (4.9227, 0.4728, 1.8353, 1.0782, 1.1370, 0.6640, 0.3410, 0.5436, 0.1704, 0.8012, 0.2265, 0.4428, 0.8012, 0.5788, 0.5000, 0.5788) was obtained after 251 function evaluations.

Table 2. Performances of Nelder and Mead Method, MDS Method and Hooke's and Jeeves Method in 16 Bar Truss Model.

	Methods		
	Nelder and Mead Method	**MDS Method**	**Hooke's and Jeeves Method**
Initial Guess	3, 2, 1, 1.5, 1, 2, 1, .5, .5, .5, .5, .5, .5, .5, .5, .5	2, 2, 2, 1, 1, 1, 1, 1, 1,1, 2, 1, 1, 1, 1, 2	5,2,5,4,1,3, 3,1,2,2,1,1, 1,1,1,1
Initial Value	660	850	1370
Function Evaluations	251	1626	896
Optimal Point	4.9227, 0.4728, 1.8353, 1.0782, 1.1370, 0.6640, 0.3410, 0.5436, 0.1704, 0.8012, 0.2265, 0.4428, 0.8012, 0.5788, 0.5000, 0.5788	4.75 0.3125, 1.25, 1, 1, 0.3750, 0.5, 0.3750, 0.25, 0.25, 0.1250, 0.25, 0.25, 0.25, 0.25, 0.25	4.722,0.2930,1.1875, 0.8906,0.9297,0.2969, 0.2969,0.2930,0.0625, 0.0625,0.1172,0.0508, 0.0508,0.0508,0.0508, 0.0508
Optimal Value	602.9846	473.1250	391.281

Better results were acquired from sixteen bar truss by applying Multi-directional search method, where the initial guess was taken in the range of 1 to 10. Many iterations were performed by taking different values in the above mentioned range and the function value in each iteration remained very close to *473.1250*. It means that this method showed the consistent performance for various initial guesses. The optimal point (4.75, 0.3125, 1.25, 1, 1, 0.3750, 0.5, 0.3750, 0.25, 0.25, 0.1250, 0.25, 0.25, 0.25, 0.25, 0.25) was obtained after 1626 function evaluations.

In the present study, continuous value field of the design variables was equally divided to discrete segment. It can be also observed from the study table 2 that even the optimal result (391.281) given by Hooke-Jeeve method was just with a weight of about 0.06% lower than the design obtained by (Atluri *et al*, 1995). While comparing with several algorithms using discrete design variable values, Hooke-Jeeve method was one of the best method. And comparing with other evolutionary algorithm, like NM method and MDS method the optimal result by Hooke-Jeeve method was with a weight about 10.9% lower than that methods.

For optimum results of sixteen bar truss problem, a general-purpose solver was required. For numerical simulation of the sixteen bar truss model, the programming environment of MATLAB was found to be quite supportive due to availability of a plenty of built-in functions. Another important advantage of MATLAB was the fact that parameters were easily settled for handling constraints as reported by (Tabassum *et al*, 2015) and (Ali *et al*, 2015).

A novel approach to solve engineering design problems based on a simple derivative free strategy was presented. The main advantage of approach was that it required a penalty function or parameters. Also, the computational cost of approach was very low. Furthermore, the proposed approach was very simple and easy to implement. Simple derivative free strategy provided better results than traditional penalty-based approaches.

Conclusion: The comparative study of three derivative free optimization methods by implementing on the formulated 16 bar truss problem is presented with an aim of identifying the capability of the three methods. These methods were implemented in MATLAB and run the program many times by taking various initial guesses and step sizes. It is observed from Table-2 that the results of Nelder – Mead are not acceptable due to its far away convergence even its number of function evaluations were smaller than the number of function evaluations of the other two methods. By comparing the optimal function values obtained by these three methods it was concluded that the performance of Hooke-Jeeves method was better than the MDS and NM methods.

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