VIBRATIONAL ANALYSIS OF SUBMERGED CYLINDRICAL SHELLS BASED ON ELASTIC FOUNDATIONS

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ABSTRACT: In this study a vibration analysis was performed of an isotropic cylindrical shell submerged in fluid, resting on Winkler and Pasternak elastic foundations for simply supported boundary condition. Love's thin shell theory was exploited for strain- and curvature- displacement relationship. Shell problem was solved by using wave propagation approach. Influence of fluid and Winkler as well as Pasternak elastic foundations were studied on the natural frequencies of submerged isotropic cylindrical shells. Results were validated by comparing with the existing results in literature.

Key words: Vibration, Submerged cylindrical shell, Love's thin shell theory, Wave propagation method, Winkler and Pasternak foundations.

INTRODUCTION

Vibrations of cylindrical shells are the most widely studied area of research because of their simple geometrical shapes. First proper shell theory was proposed by (Love, 1888). (Loy and Lam, 1997) studied the vibration of thin cylindrical shell with ring supports. The study was carried out using Sander's shell theory. The governing equations were obtained using energy functional with the Ritz method. (Iqbal *et al.,* 2009) applied wave propagation approach to analyze vibrations of functionally graded material circular cylindrical shells. (Arshad *et al*., 2010) studied vibration of bi-layered cylindrical shells with layers of different materials. Frequencies were evaluated for long, short, thick and thin cylindrical shells by varying non-dimensional geometrical parameters, length-to-radius and thickness-to- radius ratios for a simply supported boundary condition. (Arshad *et al.,* 2011) also investigated vibration analysis of bi-layered functionally graded cylindrical shells. In this case, both layers are composed of functionally graded materials and the thickness of shell layers is considered to be equal and constant. (Shah *et al*., 2010) have studied vibrations of functionally graded cylindrical shells based on elastic foundations. They amended the equations of functionally graded cylindrical shells by inducting the modulii of the Winkler and Pasternak foundations. (Shah *et al*., 2010) also presented vibration characteristics of cylindrical shells which was filled with fluid and was put on the elastic foundations. (Naeem *et al*., 2010) studied the vibration frequency characteristics of functionally graded cylindrical shells using the generalized differential Quadrature method. (Shah *et al*., 2013) calculated natural frequencies of three-layered functionally graded cylindrical shells with middle layer consisting of isotropic material resting on Winkler and Pasternak elastic foundations. (Shah *et al.,* 2013) investigated vibration characteristics of tri-layered cylindrical shells with functionally graded material middle layer based on elastic foundations for various boundary conditions.

The present study is concerned with analysis of the vibration characteristics of submerged cylindrical shells based on Winkler and Pasternak elastic foundations. Love's thin shell theory has been utilized for strain and curvature displacement relationship. The wave propagation method is used to formulate the shell eigen-frequency equation. Powerful mathematical tool MATLAB has been utilized to extract natural frequencies and mode shapes of the cylindrical shell for certain material as well as physical parameters.

Theoretical formulation: A cylindrical shell of the thin-wall was considered here as shown in Fig.1, with the geometrical parameters: length L, thickness h and mean radius R. The orthogonal coordinate system (x, θ, z) was taken to be at the surface of the shell where x, θ and z represent the axial, circumferential and radial coordinates respectively. Young's modulus E, the Poisson ratio ν and the mass density ρ were the shell material parameters. The axial, circumferential and radial displacement deformations were denoted by $u(x, \theta, t)$, $v(x, \theta, t)$ and $w(x, \theta, t)$ respectively with regard to the shell middle surface.

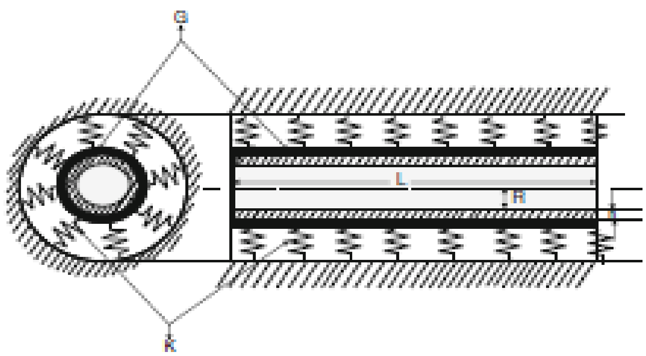

Fig. 1 Cylindrical shell on elastic foundation

The equations of motion for cylindrical shell from the Love shell theory were given in the form as:

$$\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial \theta} = \rho_t \frac{\partial^2 u}{\partial t^2} \quad (1)$$

$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial N_\theta}{\partial \theta} + \frac{2}{R}\frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R^2}\frac{\partial M_\theta}{\partial \theta} = \rho_t \frac{\partial^2 v}{\partial t^2} \quad (2)$$

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{2}{R}\frac{\partial^2 M_{x\theta}}{\partial x \partial \theta} + \frac{1}{R^2}\frac{\partial^2 M_\theta}{\partial \theta^2} - \frac{N_\theta}{R} = \rho_t \frac{\partial^2 w}{\partial t^2} \quad (3)$$

where N_x, N_θ, $N_{x\theta}$ were force resultant and M_x, M_θ, $M_{x\theta}$ were moment resultants. Which are given as:

$$\begin{bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \gamma \\ k_1 \\ k_2 \\ 2\tau \end{bmatrix} \quad (4)$$

Where e_1, e_2 and γ are the reference surface strains and k_1, k_2 and τ are the surface curvature and A_{ij}, B_{ij} and $D_{ij}(i, j = 1,2 \text{ and } 6)$ stand for extensional, coupling and bending stiffness respectively and defined as:

$$\left.\begin{aligned} A_{ij} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij}\, dz \\ B_{ij} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij}\, z\, dz \\ D_{ij} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij}\, z^2\, dz \end{aligned}\right\} \quad (5)$$

ρ_t denotes the mass density per unit length and is defined as:

$$\rho_t = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho\, dz$$

For the isotropic materials, the reduced stiffness $Q_{ij}\,(i, j = 1, 2 and\, 6)$ are given as:

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2}, \qquad Q_{12} = \frac{\nu E}{1-\nu^2}, \qquad Q_{66} = \frac{E}{2(1+\nu)} \quad (6)$$

The analysis was carried out using Love's first order thin shell theory and the relationships for strain-displacement and curvature-displacement were provided from this theory and given for a cylindrical shell as:

$$\{e_1, e_2, \gamma\} = \{\partial u/\partial x, 1/R\,(\partial v/\partial \theta + w), (\partial v/\partial x + 1/R\ \partial u/\partial \theta\} \quad (7)$$

and

$$\{k_1, k_2, \tau\} = \left\{-\frac{\partial^2 w}{\partial x^2}, -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right), -\frac{1}{R\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial v}{\partial x}\right)}\right\} \quad (8)$$

These expressions for the surface strains e_1, e_2 and γ and the curvatures k_1, k_2 and τ from the relations (7) and (8) respectively, are substituted into Eq. (4) and the resulting expressions for N_x, N_θ, $N_{x\theta}$, M_x, M_θ, $M_{x\theta}$ into equations (1)-(3), and introducing the submerged cylindrical shell, which satisfied the acoustic wave equation and the terms describing the Winkler and Pasternak foundations ($Kw - G\nabla^2 w$) in the z-direction, the equations of motion for a cylindrical shell could be written in a differential operator form as:

$$L_{11}u + L_{12}v + L_{13}w = \rho_t \frac{\partial^2 u}{\partial t^2} \quad (9)$$

$$L_{21}u + L_{22}v + L_{23}w = \rho_t \frac{\partial^2 v}{\partial t^2} \quad (10)$$

$$L_{31}u + L_{32}v + L_{33}w = \rho_t \frac{\partial^2 u}{\partial t^2} + P + Kw - G\nabla^2 w \quad (11)$$

where $L_{ij}\,(i, j = 1,2,3)$ state the differential operators with regard to x and θ and are given as

$$L_{11} = A_{11}\frac{\partial^2}{\partial x^2} + \frac{A_{66}}{R^2}\frac{\partial^2}{\partial \theta^2},$$

$$L_{12} = \frac{(A_{12} + A_{66})}{R}\frac{\partial^2}{\partial x \partial \theta} + \frac{(B_{12} + 2B_{66})}{R^2}\frac{\partial^2}{\partial x \partial \theta},$$

$$L_{13} = \frac{A_{12}}{R}\frac{\partial}{\partial x} - B_{11}\frac{\partial^3}{\partial x^3} - \frac{(B_{12} + 2B_{66})}{R^2}\frac{\partial^3}{\partial x \partial \theta^2},$$

$$L_{21} = \frac{(A_{12} + A_{66})}{R}\frac{\partial^2}{\partial x \partial \theta} + \frac{(B_{12} + B_{66})}{R^2}\frac{\partial^2}{\partial x \partial \theta},$$

$$L_{22} = \left(A_{66} + \frac{3B_{66}}{R} + \frac{2D_{66}}{R^2}\right)\frac{\partial^2}{\partial x^2} + \left(\frac{A_{22}}{R^2} + \frac{2B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^2}{\partial \theta^2},$$

$$L_{23} = \left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}\right)\frac{\partial}{\partial \theta} - \left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^3}{\partial \theta^3} - \left(\frac{B_{12} + 2B_{66}}{R} + \frac{D_{12} + 2D_{66}}{R^2}\right)\frac{\partial^3}{\partial x^2 \partial \theta},$$

$$L_{31} = -\frac{A_{12}}{R}\frac{\partial}{\partial x} + B_{11}\frac{\partial^3}{\partial x^3} + \frac{(B_{12} + 2B_{66})}{R^2}\frac{\partial^3}{\partial x \partial \theta^2},$$

$$L_{32} = -\left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}\right)\frac{\partial}{\partial \theta} + \left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^3}{\partial \theta^3} + \left(\frac{B_{12} + 2B_{66}}{R} + \frac{D_{12} + 4D_{66}}{R^2}\right)\frac{\partial^3}{\partial x^2 \partial \theta},$$

$$L_{33} = -\frac{A_{22}}{R^2} + \frac{2B_{12}}{R}\frac{\partial^2}{\partial x^2} + \frac{2B_{22}}{R^3}\frac{\partial^2}{\partial \theta^2} - D_{11}\frac{\partial^4}{\partial x^4} - 2\frac{D_{12} + 2D_{66}}{R^2}\frac{\partial^4}{\partial x^2 \partial \theta^2} - \frac{D_{22}}{R^4}\frac{\partial^4}{\partial \theta^4}, \quad (12)$$

where G stands for Pasternak elastic foundation and K for the Winkler foundation modulus. The expression for the differential operator ∇^2 is:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2}{\partial \theta^2} \quad (13)$$

The fluid exterior of cylindrical shell satisfies the acoustic wave equation which is given in cylindrical co-ordinate system (x, θ, t) as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 p}{\partial t^2} \quad (14)$$

Where P was the acoustic pressure and c was the sound speed of the fluid and t is the time.

Numerical procedure

The following modal displacement shape functions were adopted to separate the time and space variables

$$u(x, \theta, t) = A\cos(n\theta)e^{i(\omega t - k_m x)}$$

$$v(x, \theta, t) = B\sin(n\theta)e^{i(\omega t - k_m x)}$$

$$w(x, \theta, t) = C\cos(n\theta)e^{i(\omega t - k_m x)} \quad (15)$$

in the longitudinal, circumferential and transverse directions respectively. The constants A, B and C are the amplitudes of vibrations in the x, θ and z directions respectively, n is the number of circumferential waves and k_m stands for axial wave number that is associated with a boundary conditions. Wave numbers for four types of boundary conditions are given in Table 1:

Table 1 Showing wave Numbers for Different Boundry Conditions

Boundary conditions	Wave numbers
Simply supported - simply supported	$k_m = \frac{m\pi}{L}$
Clamped – clamped	$k_m = (2m+1)\pi/2$
Clamped - simply supported	$k_m = (4m+1)\pi/4$
Clamped – free	$k_m = (2m-1)\pi/2L$

These axial wave numbers k_m were selected to satisfy boundary conditions at both edges of a cylindrical shell. ω denotes the natural angular frequency for the cylindrical shell.

The associated form of the acoustic pressure field exterior of the shell, which satisfied the acoustic wave Eq. (14) is given as:

$$P = P_m \cos(n\theta) H_n^{(2)}(k_r r)\, e^{i(\omega t - k_m x)} \quad (16)$$

where P_m was the pressure amplitude, $H_n^{(2)}(\;)$ was the second kind Hankel function of order *n*. The radial wave number k_r and axial wave number k_m are related by a usual vector relation $k_r = (k_0 - k_m)^{\frac{1}{2}}$, where $k_0 = \omega$ /c is the fluid acoustic wave number. In order to ensure that the acoustic fields satisfy the conditions, usually $k_r = (k_0 - k_m)^{\frac{1}{2}}$ is taken when $k_0 \geq k_m$ and $k_r = -i(k_m - k_0)^{\frac{1}{2}}$ is chosen for $k_0 < k_m$.

To ensure that fluid remains in contact with shell wall, the fluid radial displacement must be equal to the shell radial displacement at the interface of the outer shell wall and the fluid. This coupling condition is:

$$-\left(\frac{1}{i\omega\rho_f}\right)\left(\frac{\partial P}{\partial r}\right)_{r=R} = \left(\frac{\partial w}{\partial t}\right)_{r=R} \quad (17)$$

which gives:

$$P_m = \left[\frac{\omega^2 \rho_f}{k_r H_n^{(2)'}(k_r R)}\right] C \quad (18)$$

where ρ_f is the fluid density and $H_n^{'(2)}(k_r R)$ denotes differentiation with respect to the argument $k_r R$.

Making substitution for the displacement functions u, v and w from the expression (15) in system of equations (9-11) and simplifying the algebraic expression and rearranging the terms, the frequency equation is written in the following eigenvalue form:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ -C_{12} & C_{22} & C_{23} \\ -C_{13} & C_{23} & C_{33} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \omega^2 \begin{bmatrix} \rho h & 0 & 0 \\ 0 & \rho h & 0 \\ 0 & 0 & \rho h - \frac{\rho_f}{k_r} \frac{H_n^{(2)}(k_r)}{H_n^{'(2)}(k_r)} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

(19)

where $C_{ij}\ (i, j = 1, 2, 3)$ are coefficients of stiffness matrix and their values are given below:

$$C_{11} = k_m^2 A_{11} + n^2 \frac{A_{66}}{R^2}$$

$$C_{12} = ink_m \frac{A_{12} + A_{66}}{R} + \frac{2B_{66} + B_{12}}{R^2}$$

$$C_{13} = i\, k_m \left(\frac{A_{12}}{R} + B_{11} k_m^2 + n^2 \frac{B_{12} + 2B_{66}}{R^2}\right)$$

$$C_{22} = n^2 \left(A_{22}/R^2 + (2B_{22})/R^3 + D_{22}/R^4\right) + k_m^2 \left(A_{66} + \frac{4B_{66}}{R} + \frac{4D_{66}}{R^2}\right)$$

$$C_{23} = n\left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3} + n^2\left(\frac{B_{22}}{R^3} + D_{22}/R^4\right) + k_m^2\left(\frac{B_{12} + 2B_{66}}{R} + \frac{D_{12} + 4D_{66}}{R^2}\right)\right)$$

$$C_{33} = \frac{A_{22}}{R^2} + \frac{2k_m^2}{R} B_{12} + 2n^2 \frac{B_{22}}{R^3} + D_{11} k_m^4 + 2n^2 k_m^2 \left(\frac{D_{12} + 2D_{66}}{R^2}\right) + n^4 \frac{D_{22}}{R^4} + K + G\left(k_m^2 + \frac{n^2}{R^2}\right) \quad (20)$$

Equation (20) is solved for shell frequencies and mode shapes using computer software MATLAB. The three frequencies were obtained corresponding to the axial, circumferential and radial displacements.

RESULTS AND DISCUSSION

To show the validity and accuracy of the present approach, results were compared with the existing results of the literature. In this regard, a comparison of the variation of the natural frequencies (Hz) of the isotropic cylindrical shell with simply supported-simply supported end condition was taken in Table 2, against axial half wave number *m*, for the shell parameters *L*= 8 in*, R*=2 in, *h*=0.1 in, *E*= 30×10^6 lbf in^{-2}, ν =0.3, ρ =7.35$\times 10^{-4}$ lbf$s^2 in^{-4}$ with the results of (Warburton et al., 1961). Table 3 showed comparisons of the values of natural frequencies (Hz) of an isotropic cylindrical shell with those results evaluated by (Goncalves *et al*. 2006) with simply supported - simply supported boundary conditions. The comparisons were carried out for the parameters: *L = 0.41m, R=0.3015 m, h=0.001m*, *E*= 2.1×10^{11} N/m², ν =0.3, ρ =7850 kg/m^{3}for the axial mode m=1. It was seen from the two sets of results that the present frequencies were a little bit lower than those evaluated by Goncalves *et al.* This difference was because of the two different approaches used in above reference and the present study. Table 4 shows the comparison of the variation of frequency parameter of isotropic cylindrical shells with circumferential wave number *n* to that of (Li. 2008) for simply-supported end condition having shell parameters (*m* = 1, *L/R* = 20, *h/R*=0.05). In table 5, a comparison of the coupled natural frequencies (Hz) has been taken of a clamped-clamped isotropic cylindrical shell for different pattern of mode shapes (*m, n*) with the evaluated results of Finite element method / Boundary element method and (SYSNOISE, 1998) for shell geometrical parameters *L=20,R=1, h=0.01*. In Table 6 a comparison has been made of the dimensionless frequency parameter Ω for an isotropic cylindrical shell resting on Winkler foundation having shell parameters *h/R*=0.01, *L/R*=2, K=10^{-4} N/m^3 with the already evaluated results of the (Sofiyev et al. 2010) for simply supported-simply supported boundary conditions.

These Tables 2-6, show a good agreement of the results obtained from the present approach and the other numerical techniques for the solution of the shell problem.

Table 2 Showing comparison of natural frequency (Hz) for an isotropic cylindrical shell with simply supported-simply supported end condition with shell parameters *L*= 8 in, *R*=2 in, *h*=0.1 in, *E*= 30$\times 10^6$ lbf in^{-2} , ν =0.3, ρ =7.35$\times 10^{-4}$ lbf$s^2 in^{-4}$

m	Warburton (1961)	Present
1	2199.3	2194.4
2	4041.9	4031.1
3	6620.0	6605.9
4	9124.0	9108.4
5	11357.0	11343.4
6	13384.0	13374.9

Table 3 Showing comparison of natural frequencies (Hz) for an isotropic cylindrical shell (*L = 0.41m, R=0.3015 m, h=0.001m, E= 2.1* $\times 10^{11}$ N/m², ν =0.3, ρ =7850 kg/m^3

n	Goncalves *al. et* (2006)	Present	Difference %
7	303.35	301.93	0.47
8	280.94	278.99	0.69
9	288.71	286.37	0.81
10	318.40	315.83	0.81
11	363.33	360.64	0.74
12	419.19	416.44	0.66
13	483.51	480.74	0.57
14	554.97	552.22	0.50

Table 4: Showing comparison of frequency parameters Ω =*ωR*√[(1 − *v*2)*ρ/E*] for an isotropic cylindrical shell with simply supported boundary condition (*m* = 1, *L/R* = 20, *h/R*=0.05).

n	Li (2008)	Present
0	0.09295	0.092968
1	0.01610	0.016102
2	0.03930	0.039271
3	0.109824	0.109811
4	0.210284	0.210277

Table 5 Showing comparison of coupled natural frequencies (Hz) of a clamped-clamped cylindrical shell submerged in water by FEM/BEM, SYSNOISE with the natural frequencies of present method (*L=20,R=1, h=0.01*)

n	FEM/BEM (1998)	SYSNOISE (1998)	Present	Modal Shape (m, n)
1	4.92	5.00	4.95	(1, 2)
2	9.06	9.62	8.93	(1, 3)
3	10.71	11.22	10.61	(2, 3)
4	11.24	11.39	11.67	(2, 2)
5	14.70	15.18	14.57	(3, 3)
6	18.68	20.58	18.25	(1, 4)
7	19.14	20.96	18.70	(2, 4)
8	20.37	22.10	19.90	(3, 4)

Table 6: Showing comparison of the dimensionless frequency parameter Ω for a cylindrical shell resting on Winkler elastic foundation (*h/R*=0.01, *L/R*=2, K=10^{-4} N/m^3) for simply supported-simply supported boundary conditions.

n	Sofiyev et al. (2010)	Present
1	0.6792	0.6812
2	0.3646	0.3661
3	0.2080	0.2090
4	0.1342	0.1351

In Fig. 2, variation of natural frequencies (Hz) of an isotropic empty cylindrical shell have been compared before and after submerged against circumferential wave number *n*, for simply supported-simply supported end condition on elastic foundations and geometrical and material parameters as given in the caption of this figure.

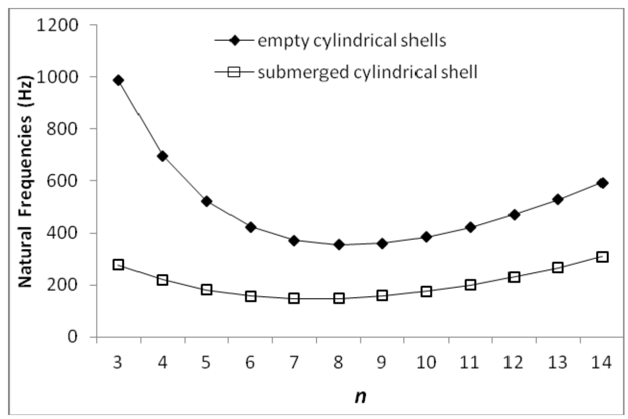

Fig. 2 Comparison of the variation of natural frequencies (Hz) of empty and submerge isotropic cylindrical shells against circumferential wave number *n*, for simply supported-simply supported end condition having shell parameters *(m=1, L=0.41, R=0.3015, h=0.001, E=2.1*$\times 10^{11}$ *N/m*^{2} *,v=0.3,* ρ *=7850kg/m*^{3} *,G=0,K=1.5*$\times 10^7$ *)*

It was noticed that the natural frequencies (Hz) of both kind of cylindrical shells behave alike, i.e. it first decrease up to circumferential wave number *n* = *8*, and then it increased with the increasing values of *n*. The difference of the natural frequencies (Hz) of both kinds of cylindrical shells decrease up to *n=8* and for *n* > *8*, this difference again start to increase. This concluded that the natural frequencies of the shells decreased significantly up to nearly three times, when it was submerged in fluid.

In Fig. 3, a comparison of the variation of natural frequencies (Hz) of an empty isotropic cylindrical shell before submerged has been taken against circumferential wave number *n*, for different values of Winkler foundation K==1.5$\times 10^7$, 2.5$\times 10^7$ and Pasternak *G = 0*, having shell parameters given in its caption. It was observed by changing Winkler foundation K, the natural frequencies (Hz) of both kinds of shells behave alike, i.e. it first decrease up to circumferential wave number *n = 8*, and for higher value of circumferential wave number n > 8, both kinds of frequencies initiate to increase. For lower and higher values of circumferential wave number *n*, both types of frequencies seem to converge. Difference between both types of natural frequencies was not considerable. Rate of decrement was faster than rate of increment of both kinds of the natural frequencies.

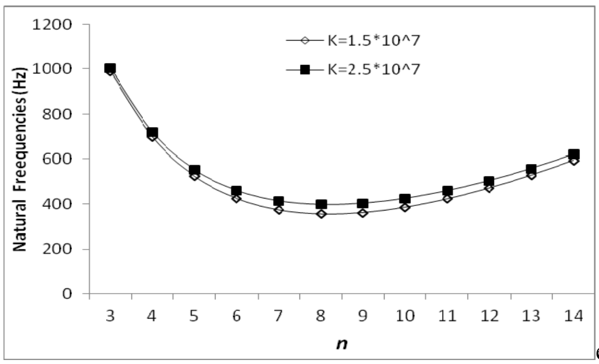

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Fig. 3 Comparison of the variation of natural frequencies (Hz) of empty isotropic cylindrical shells against circumferential wave number *n*, for different values of Winkler foundation for simply supported-simply supported end condition (*m=1, L=0.41, R=0.3015, h=0.001, E=2.1*$\times 10^{11}$ *N/*m^2*,v=0.3,* ρ *=7850kg/*m^3*,G=0*)

In Fig. 4, a variation of natural frequencies (Hz) of an empty isotropic cylindrical shell after submersion has been taken against circumferential wave number *n*, for different values of Winkler foundation *K=1.5*$\times 10^7$, *2.5*$\times 10^7$ and Pasternak foundation *G = 0*, having shell parameters given in its caption. It was seen that by changing Winkler foundation *K*, the natural frequencies (Hz) of both kinds of shells behave alike, i.e. it first decrease up to circumferential wave number *n = 8*, and for higher value of circumferential wave number *n* > 8, both kinds of frequencies began to increase. For lower and higher values of circumferential wave number *n*, both types of frequencies seem to converge. Difference between both types of natural frequencies was not considerable. Moreover rate of decrement was slower than rate of increment of both kinds of the natural frequencies. Natural frequencies of both kinds of shells have moon type figure which was concave up and whose edges were sharp, thin and whose middle part was thick.

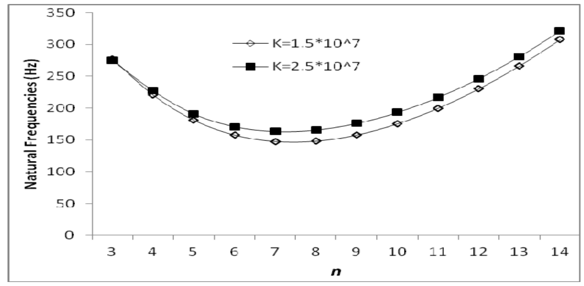

Fig. 4 Comparison of the variation of natural frequencies (Hz) of submerged isotropic cylindrical shells against circumferential wave number *n*, for different values of Winkler foundation for simply supported-simply supported end condition (*m=1, L=0.41, R=0.3015, h=0.001, E=2.1*$\times 10^{11}$ *N/*m^2*, v=0.3,* ρ *=7850kg/*m^3*,G=0*)

In Fig. 5, a comparison of the variation of natural frequencies (Hz) of an empty isotropic cylindrical shell before submersion has been sketched against circumferential wave number n, for different values of Pasternak foundation G =1.5$\times 10^7$, 2.5$\times 10^7$ and Winkler *K = 0*, having shell parameters given in its caption.

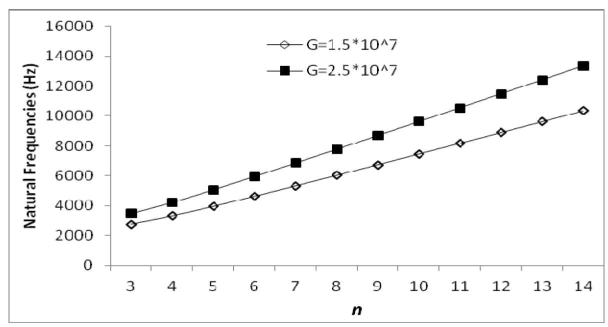

Fig. 5 Comparison of the variation of natural frequencies (Hz) of empty isotropic cylindrical shells against circumferential wave number *n*, for different values of Pasternak foundation for simply supported-simply supported end condition (m=*1, L=0.41, R=0.3015, h=0.001*, *E*=2.1$\times 10^{11}$N/m^2, v=0.3, ρ =7850kg/m^3,K=0)

It was observed by changing Winkler foundation K, the natural frequencies (Hz) of both kinds of shells behave alike, i.e. they increased with the circumferential wave number *n* for both kinds of cylindrical shells. For lower and higher values of circumferential wave number *n*, both types of frequencies seem to converge but for higher value if circumferential wave number *n*, they have diverging behaviour. Difference between both types of natural frequencies was substantial. Rate of increment of the frequencies for higher G was higher than that of lower of values of G. Natural frequencies of both kinds of shells depart sharply for the increasing values of circumferential wave number *n*,

Similar behaviour of the natural frequencies (Hz) has been observed for the empty cylindrical shells after submersion for different values of Pasternak foundation, shown in Fig. 6 but in this case natural frequencies of the shell were lower down remarkably as compared to the shell before submerged. For lower values of circumferential wave number *n*, both types of frequencies seem to converge. Slightly bending behaviour of the natural frequencies has been observed for both kinds of submerged shells.

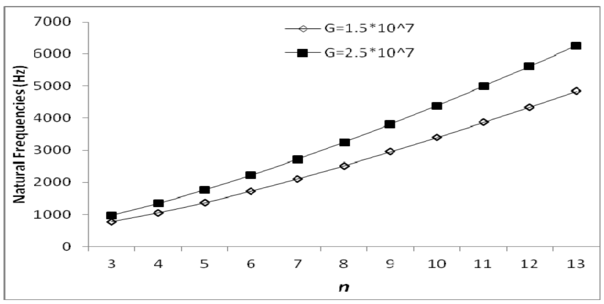

Fig. 6 Comparison of the variation of natural frequencies (Hz) of submerged isotropic cylindrical shells against circumferential wave number *n*, for different values of Pasternak foundation for simply supported-simply supported end condition (*m=1, L=0.41, R=0.3015, h=0.001, E=2.1*$\times 10^{11}$ *N/*m^2*, v=0.3,* ρ *=7850kg/*m^3*,K=0*)

In Fig. 7, a comparison of the variation of natural frequencies (Hz) of an empty isotropic cylindrical shell before submersion has been drawn against circumferential wave number *n*, for different values of Pasternak foundation G =1.5$\times 10^7$, 2.5$\times 10^7$ along Winkler foundations *K* = 1$\times 10^7$, 1.5$\times 10^7$ respectively, having shell parameters given in its caption. It was observed by changing Pasternak G and Winkler foundations *K*, the natural frequencies (Hz) of both kinds of shells behave alike, i.e. they increased with the circumferential wave number *n* for both kinds of cylindrical shells. For lower values of circumferential wave number *n*, both types of frequencies appear to converge but for higher value if circumferential wave number *n*, they have diverging response. Difference between both types of natural frequencies was substantial. Rate of increment of the frequencies for higher G and K is higher than that of lower of values of G and K. Natural frequencies of both kinds of shells apart sharply for the increasing values of circumferential wave number *n*.

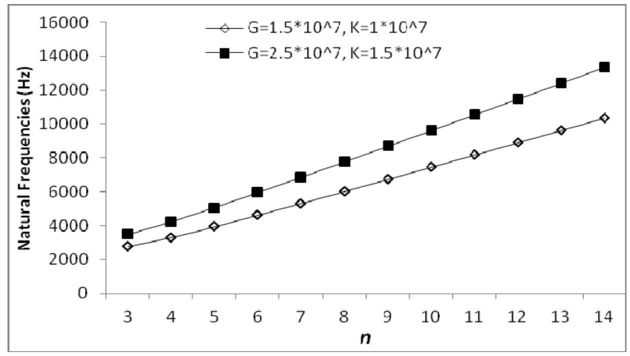

Fig. 7 Comparison of the variation of natural frequencies (Hz) of empty isotropic cylindrical shells against circumferential wave number *n*, for different values of Pasternak and Winkler foundations for simply supported-simply supported end condition (*m=1, L=0.41, R=0.3015, h=0.001, E=2.1*$\times 10^{11}$ *N/*m^2*, v=0.3,* ρ *=7850kg/*m^3)

Alike behaviour of the natural frequencies (Hz) have been observed for the empty cylindrical shells after submersion for different values of Pasternak and Winkler foundations, revealed in Fig. 8 but in this case natural frequencies of the shell was lower down remarkably as compared to the shell before submersion. For lower values of circumferential wave number *n*, both types of frequencies seem to converge but for higher values of circumferential wave number n they kept diverging approach. Faintly bending behaviour of the natural frequencies was observed for both kinds of submerged shells having different elastic foundations.

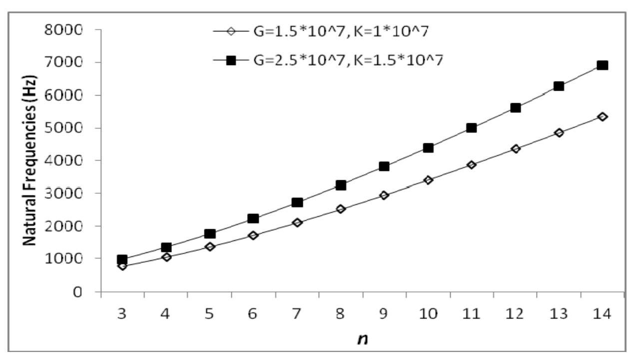

Fig. 8 Comparison of the variation of natural frequencies (Hz) of submerged isotropic cylindrical shells against circumferential wave number *n*, for different values of Pasternak and Winkler foundations for simply supported-simply supported end condition (*m=1, L=0.41, R=0.3015, h=0.001, E=2.1*$\times 10^{11}$ *N/*m^2*, v=0.3,* ρ *=7850kg/*m^3)

Conclusion: In the present analysis natural frequencies of an isotropic cylindrical shell submerged in fluid for different pattern of Winkler and Pasternak elastic foundations was carried out for simply supported boundary condition. Influence of the fluid in which an empty cylindrical shell was submerged was found to be significant, whereas Winkler and Pasternak elastic foundations also affect the natural frequencies of submerged isotropic cylindrical shells. Pasternak foundations influence more than that of Winkler foundations. This work could be extended to study vibration characteristics of fluid-filled cylindrical shell submerged in fluid on elastic foundations. MATLAB computer software has been used to obtain significant results for this study.

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