REDUCED DIFFERENTIAL TRANSFORM METHOD FOR COUPLED PROBLEM OF THE SYMMETRIC REGULARIZED LONG WAVE EQUATIONS

S. Shafi, *M. S. Iqbal, M. I. Qadir

Department of Mathematics, University of Engineering and Technology, Lahore, Pakistan.
* Department of Mathematics, Virtual University of Pakistan, Lahore, Pakistan.
Corresponding author e-mail: sshafiae@gmail.com

ABSTRACT: The aim of this research was to deal with coupled problem of the symmetric regularized long wave equations. A semi analytical approach namely, the reduced differential transform method was applied to find the numerical solution of the coupled problem of symmetric regularized long wave equations. Mainly this technique was formed to compute the approximate solution in the form of convergent power series with simple and easy calculating procedures. The numerical results were computed by using Wolfram Mathematica 9.0 which determined the efficacy and pertinence of the proposed method. The obtained numerical results were compared with the exact solution which verified the recommended methodology. It was concluded that the approximate results were very much stable and convergent to the exact solution for the coupled problem of the symmetric regularized long wave equations.

Key words: reduced differential transform method, symmetric regularized long wave, coupled problem, and mixed partial derivatives.

(Received 20-07-2016 Accepted 10-11-2016)

INTRODUCTION

The concept of nonlinear partial differential equation is not new rather it is centuries old. It has been applied to explore new research dimensions in the second half of the 20th century. The theory of nonlinear partial differential equations is one of the important research areas in different fields. Countless problems appear in various fields of engineering, mathematics and physics such as electrodynamics, fluid flow and quantum mechanics. One of the major roles of nonlinear partial differential equations is the study of nonlinear wave equations. For example, the Benjamin-Bona-Mahony equation is a nonlinear wave equation, which is also known as the regularized long wave equations (RLWE). The nonlinear wave equation with unbounded domain and several other such equations gains a lot of attention of scientists and engineers (Debnath, 1997). There are numerous numerical methods and techniques which have been applied to the initial and boundary value problems (IBVP) depending upon the nature of problems. For solving linear and nonlinear initial value problems in electrical circuits, the one-dimensional differential transform method (DTM) is used (Zhou, 1986). This approach is based on the Taylor series and hence builds a semi analytical technique. IBVP is transformed into a recurrence relation by using DTM. One-dimensional DTM is convenient for obtaining approximate solution of a problem that converges to the exact solutions. One-dimensional DTM can be extended (Chen and Ho, 1999) into two-dimensional DTM for resolving the integral and differential equations. DTM is basically a semi analytical method which is used to get numerical solutions of linear and nonlinear differential equations. In DTM, each variable is required to be transformed that creates complexity in computations. This difficulty is overcome by the reduced differential transform method (RDTM) by making some changes (Keskin and Oturanc, 2010 a). Instead of transforming all the variables, this proposed method targets the transformation of the one variable calling the domain of interest (Keskin and Oturanc, 2009).

RDTM is a technique that can be applied to work out for coupled problem of the symmetric regularized long wave equations (SRLWE). A closely related problem of single equation has been solved by this technique (Keskin and Oturanc, 2010 b). This technique is proposed to model the dynamics of weakly nonlinear ion acoustic and space charge wave’s and is formulated as for coupled problem of SRLWE. The name, SRLWE was given due to its resemblance to RLWE (Seyler and Fenstermacher, 1984).

SRLWE has been explored by using the conservative finite difference schemes (Wang *et al*., 2007). Recently in 2014, research work on SRLWE has been done by conservative Crank-Nicolson finite difference scheme (Hu *et al*., 2014).

The present study is aimed to resolve the coupled problem of symmetric regularized long wave equations by the reduced differential transform method. The key idea of a long wave is the transformation of data for longer distances in shorter time. The most of the problems dealt in the context of RDTM did not deal with the coupled problem when there are mixed partial derivatives. The current study deals with mainly such type of implications of partial differential equations. Finally, the numerical result obtained is compared with the exact solution and implementation of the method is verified as well.

MATERIALS AND METHODS

The motivation of this study was to solve the coupled problem of symmetric regularized long wave equations. An approximate approach, namely, the reduced differential transform method was applied to obtain numerical solution. Basically, this method was a semi analytical technique to achieve approximate solution.

By using RDTM, the following coupled problem of SRLWE was considered (Seyler and Fenstermacher, 1984), is presented below

$$\left.\begin{aligned} \partial_t z + \partial_x \varphi + z\partial_x z - \partial_{xxt} z = 0 \\ \partial_t \varphi + \partial_x z = 0 \end{aligned}\right\}, \qquad (1)$$

where z was meant for the fluid velocity and φ denoted the dimensionless electron charge density.

The solitary wave solution of the coupled problem (1) was presented below (Seyler and Fenstermacher, 1984),

$$\left.\begin{aligned} z(x,t) &= \frac{3(\nu^2-1)}{\nu} sech^2 \frac{1}{2}\sqrt{\frac{\nu^2-1}{\nu^2}}(x-\nu t) \\ \varphi(x,t) &= \frac{3(\nu^2-1)}{\nu^2} sech^2 \frac{1}{2}\sqrt{\frac{\nu^2-1}{\nu^2}}(x-\nu t) \end{aligned}\right\}, \qquad (2)$$

where ν indicated the velocity such that $\nu^2 > 1$.

In this paper, the coupled problem of SRLWE (1) was studied with the following initial condition:

$$\left.\begin{aligned} & z(x,0) = z_0(x), \\ & \varphi(x,0) = \varphi_0(x), \quad x \in [x_L, x_R] \end{aligned}\right\}, \qquad (3)$$

where $z_0(x)$ and $\varphi_0(x)$ were two known smooth functions which converged to zero rapidly as $|x| \to \infty$.

i.e. $z(x,t) \to 0,\ \varphi(x,t) \to 0$ as $|x| \to \infty, t > 0$.

The main idea of a long wave was the transformation of data for longer distances in shorter time.

If the function is $z(x)$ then its differential transformation is given by the relation $Z(k) = \frac{1}{k!}\left(d_x^k z(x)\right)_{x=0}$ which showed that the variable x and all ordinary derivatives of $z(x)$ were represented by a parameter k. Similarly by the differential transformation, the function $z(x,t)$ depending on two variables (x,t) could also be transformed as

$$Z(h,k) = \frac{1}{h!\,k!}\left(\partial_{xt}^{h+k} z(x,t)\right)_{(0,0)} = \frac{1}{h!}.\frac{1}{k!}\left(\partial_x^h \partial_t^k z(x,t)\right)_{(0,0)},$$

where both variables x and t as well as all partial derivatives with respect to x and t could be represented by two parameters h and k respectively. When all the independent variables and derivatives with respect to them were involved in transformation to the corresponding parameters, in that case such transformation was known as differential transformation.

It was worth noticing that if a variable of interest (domain of interest) was selected on the function $z(x,t)$ and t was supposed as a variable of interest then it carried out the transformation with respect to the variable t only, it had the following formula:

$$z(x,k) = \frac{1}{k!}\left(\partial_t^k z(x,t)\right)_{t=0}.$$

A re-writing of the above equation was as follows:

$$Z_k(x) =: \frac{1}{k!}\left(\partial_t^k z(x,t)\right)_{t=0}. \qquad (4)$$

Now the function $z(x,t)$ was transformed in $Z_k(x)$, this transformation was the reduced differential transformation. Here, $Z_k(x)$ represented the transformed function and $z(x,t)$ represented the original function. Further, if it was reported that function $z(x,t)$ was transformed into the function $Z_k(x)$ then all the information about t and all partial derivatives with respect to t could be expressed from the index k of $Z_k(x)$, that was why, this function $Z_k(x)$ was written as t-dimensional spectrum function of $z(x,t)$.

The derived result was assumed by the following inverse transformation reported by (Abazari *et al*., 2013; Ayaz, 2004 and Tari *et al*., 2009).

$$z(x,t) = \sum_{k=0}^{\infty} Z_k(x) t^k. \qquad (5)$$

From (4) and (5), it was obtained below reported by (Keskin and Oturanc, 2010 a):

$$z(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\partial_t^k z(x,t)\right)_{t=0} t^k. \qquad (6)$$

According to (6), it was observed that the idea of RDTM was initiated from DTM.

From (4) and (6), the following relations were found and is tabulated below reported by (Cenesiz *et al*., 2010; Keskin and Oturanc, 2010c and Rawashdeh, 2013).

Table 1: some relations of the reduced differential transform method.

Original Function	Transformed Function
$z(x,t)$	$Z_k(x)=\frac{1}{k!}\left(\partial_t^k z(x,t)\right)_{t=0}$
$z(x,t)=g(x,t)\pm h(x,t)$	$Z_k(x)=G_k(x)\pm H_k(x)$
$z(x,t)=\alpha g(x,t)$, where α is a constant	$Z_k(x)=\alpha G_k(x)$
$z(x,t)=x^m t^n$	$Z_k(x)=x^m\delta(k-n);\ \delta(k)=\begin{cases}1; & k=n\\ 0; & k\neq n\end{cases}$
$z(x,t)=x^m t^n g(x,t)$	$Z_k(x)=x^m G_{k-n}(x)$
$z(x,t)=g(x,t)\,h(x,t)$	$Z_k(x)=\sum_{r=0}^{k} H_r(x)G_{k-r}(x)=\sum_{r=0}^{k} G_r(x)H_{k-r}(x)$
$z(x,t)=\partial_x^n g(x,t)$	$Z_k(x)=D_x^n G_k(x)$
$z(x,t)=\partial_t^r g(x,t)$	$Z_k(x)=(k+1)(k+2)(k+r)G_{k+1}(x)=\frac{(k+1)!}{k!}G_{k+r}(x)$

The proposed method could be briefly described by considering simple IBVP, $\partial_t z+\partial_{xx} z=x$, with initial condition i.e. $u(x,0)=0$, and boundary conditions i.e. $u(0,\,t)=0,\ u(1,t)=t$. Its exact solution was $u(x,t)=x\,t$.

By applying the relations from the Table-1 on the above IBVP, it was obtained as given below,

$$Z_{k+1}(x)=\frac{1}{(k+1)}\left(x-\partial_{xx}Z_k(x)\right),\text{ and } Z_0(x)=0.$$

Now for $k=0,1,2,\ldots$, the IBVP deduced that

$$Z_1(x)=x,$$

$$Z_2(x)=\frac{1}{2}x,$$

$$Z_3(x)=\frac{1}{3}x,\ldots$$

Finally, the solution of IBVP was found to be

$$z(x,\,t)=\sum_{k=0}^{\infty} Z_k(x)t^k=Z_0(x)\,t^0+Z_1(x)\,t^1+Z_2(x)\,t^2+...$$

$$=Z_0(x)+Z_1(x)\,t+Z_2(x)\,t^2+...$$

$$=xt+\frac{1}{2}xt^2+\frac{1}{3}xt^3+...$$

Cleary, this series (numerical) solution obtained by RDTM converged to exact solution for sufficiently small 't'.

When the Table-1 was applied on the nonlinear term of coupled problem of SRLWE (1), then the reduced differential transformation of nonlinear term i.e. $z\partial_x z$ was as under

$$z\partial_x z=\sum_{r=0}^{k}\partial_x Z_r(x)Z_{k-r}(x),$$

$$=\sum_{r=0}^{k}Z_{k-r}(x)\partial_x Z_r(x).$$

And the reduced differential transformation of mixed partial derivatives term of SRLWE (1) was

$$\partial_{xxt}z=\partial_{xxt}\left(\tilde{z}_k(x,t)\right)\text{ (say)},$$

because $z(x,t)=\sum_{k=0}^{\infty}Z_k(x)t^k$.

The finite terms approximation of $z(x,t)$ was reported by (Keskin and Oturanc, 2010 d) is given below

$$\tilde{z}_n(x,t)=\sum_{k=0}^{n}Z_k(x)t^k,$$

where n was the order of approximate solutions which led to

$$\partial_{xxt}z=\partial_{xxt}\left\{\sum_{n=0}^{k}Z_n(x)t^n\right\}.$$

The iteration formula for coupled problem of SRLWE (1) was

$$\left.\begin{aligned}Z_{k+1}(x)&=-\frac{1}{k+1}\begin{bmatrix}\partial_x\Phi_k(x)+\sum_{r=0}^{k}Z_{k-r}(x)\partial_x Z_r(x)\\ -\partial_{xxt}\sum_{n=0}^{k}Z_n(x)t^n\end{bmatrix}\\ \Phi_{k+1}(x)&=-\frac{1}{k+1}\left[\partial_x Z_k(x)\right]\end{aligned}\right\},\quad(7)$$

The equation (3) could be transformed as,

$$z_0(x)=Z_0(x)\text{ and }\varphi_0(x)=\Phi_0(x).\quad(8)$$

For $k = 0,1,2,3,\ldots$ in (7), different values of $Z_k(x)$ and $\Phi_k(x)$ could be attained.

By setting (8) into (7), the solution of $Z_k(x)$ and $\Phi_k(x)$ is given below

$$\left.\begin{aligned} z(x,t) &= \sum_{k=0}^{\infty} Z_k(x)t^k \\ \varphi(x,t) &= \sum_{k=0}^{\infty} \Phi_k(x)t^k \end{aligned}\right\}. \qquad (9)$$

By using the above transformed formulation, we obtained the desired series solution that could be estimated by the suitable programming.

In the next section, the above Table-1 was applied in examples to solve and illustrate the effectiveness plus accuracy of the method. Wolfram Mathematica 9.0 was used for the numerical evaluation of the coupled problem of SRLWE (1).

RESULTS AND DISCUSSIONS

To evaluate the execution and precision of the reduced differential transform method for resolving the coupled problem of SRLWE (1), the problem was analyzed and hundreds of numerical tables were formed at various time intervals. Since the long waves gave the inference about the transformation for long distances in short time. The solution of the problem was stable and the result was very much physical for the long distances along with short time. Wolfram Mathematica 9.0 was used for numerical computation. Some of the findings are presented here as examples and are given below:

Example 1: While considering the coupled problem of SRLWE (1) with the initial conditions is given as under:

$$\left.\begin{aligned} z(x,0) &= z_0(x) = \frac{5}{2}\sec h^2 \frac{\sqrt{5}}{6}x \\ \varphi(x,0) &= \varphi_0(x) = \frac{5}{3}\sec h^2 \frac{\sqrt{5}}{6}x \end{aligned}\right\}. \qquad (10)$$

By applying the Table-1 on (10), the reduced differential transformation was as under

$$\left.\begin{aligned} Z_0(x) &= \frac{5}{2}\sec h^2 \frac{\sqrt{5}}{6}x \\ \Phi_0(x) &= \frac{5}{3}\sec h^2 \frac{\sqrt{5}}{6}x \end{aligned}\right\}. \qquad (11)$$

The following tables have shown the comparison of the numerical results of the coupled problem of SRLWE (1) with the exact solution (2).

The comparison of RDTM solution of coupled problem of SRLWE (1) with the exact solution $z(x,t)$ of (2) at $t = 0.3$ and $t = 0.1$ is given below.

Table 2: Comparison of RDTM solution of coupled problem of SRLWE (1) with the exact solution $z(x,t)$ of (2) at $t = 0.3$ and $t = 0.1$.

			at $t = 0.3$		
x	**Exact Solution**	**Three Terms RDTM**	**Absolute Error**	**Four Terms RDTM**	**Absolute Error**
0	2.430985151677	2.221354166666	0.209630985010	2.221354166666	0.209630985010
2	1.821512232731	2.058404841027	0.236892608295	2.072598728014	0.251086495282
4	0.618480322493	0.583385609749	0.035094712744	0.587029978924	0.031450343568
6	0.154766415941	0.133754155121	0.021012260820	0.134037659587	0.020728756353
8	0.035720329401	0.030188169806	0.005532159595	0.030227286529	0.005493042872
10	0.008089531113	0.006801510534	0.001288020579	0.006809071745	0.001280459368
			at $t = 0.1$		
0	2.492203747266	2.469039351851	0.023164395414	2.469039351851	0.023164395414
2	1.607092030134	1.664439701504	0.057347671369	1.664965401022	0.057873370887
4	0.507948171867	0.496418901304	0.011529270563	0.496553877940	0.011394293926
6	0.124539301446	0.118338932109	0.006200369336	0.118349432275	0.006189869171
8	0.028604106928	0.026980249644	0.001623857284	0.026981698411	0.001622408516
10	0.006470726842	0.006092911875	0.000377814966	0.006093191920	0.000377534921

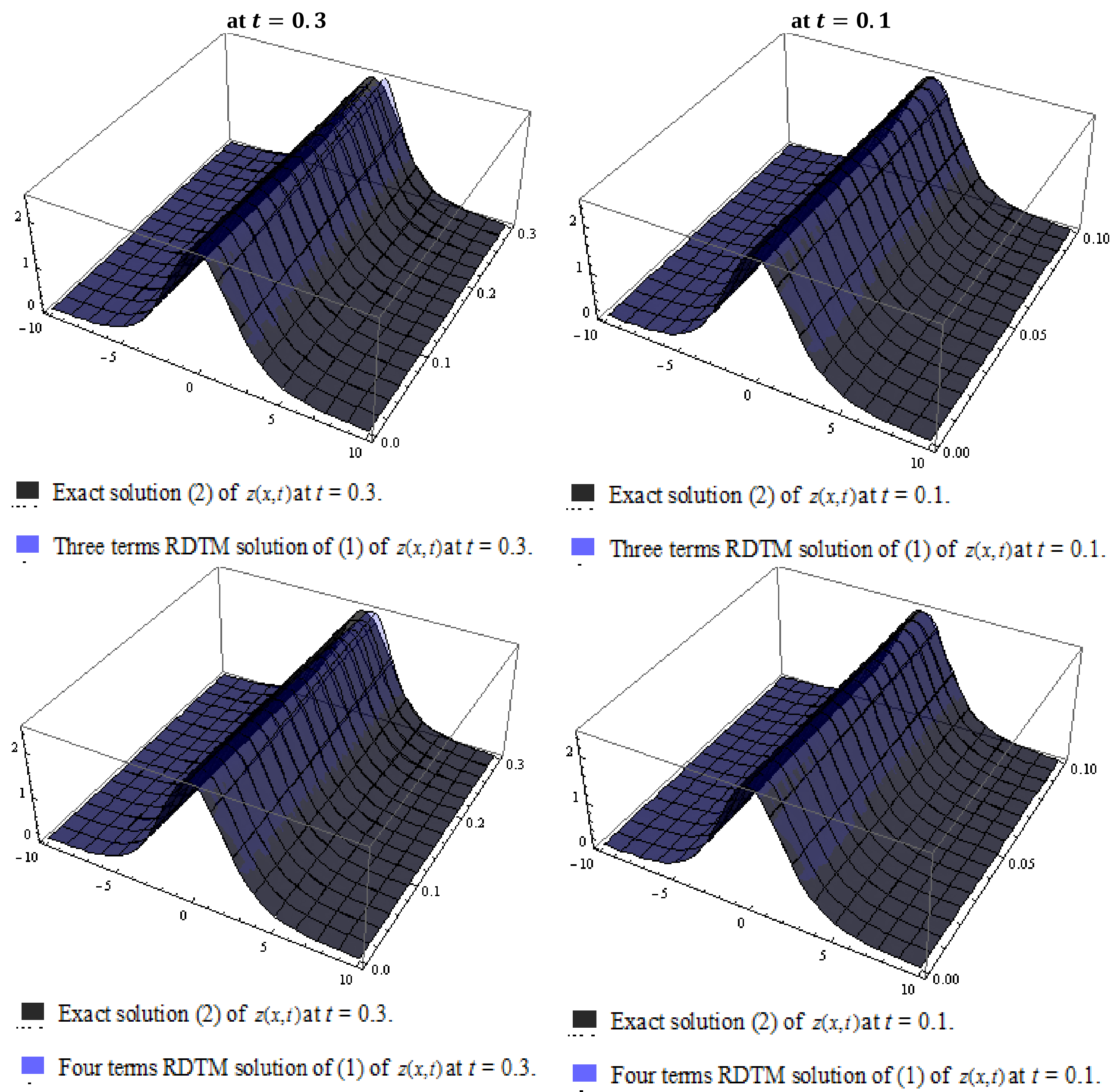

Fig 1: Comparison of RDTM solution of coupled problem of SRLWE (1) with the exact solution $z(x,t)$ of (2) at $t = 0.3$ and $t = 0.1$.

The comparison of RDTM solution of coupled problem of SRLWE (1) with the exact solution $\varphi(x,t)$ of (2) at $t = 0.3$ and $t = 0.1$ is given below

Table 3: Comparison of RDTM solution of coupled problem of SRLWE (1) with the exact solution $\varphi(x,t)$ of (2) at $t = 0.3$ and $t = 0.1$.

		at $t = 0.3$			
x	**Exact Solution**	**Three Terms RDTM**	**Absolute Error**	**Four Terms RDTM**	**Absolute Error**
0	1.620656767784	1.567708333333	0.052948434451	1.567708333333	0.052948434451
2	1.214341488487	1.242937730918	0.028596242430	1.243018809073	0.028677320585

4	0.412320214995	0.412766631579	0.000446416583	0.414783933584	0.002463718589
6	0.103177610627	0.101171885693	0.002005724934	0.101424729815	0.001752880812
8	0.023813552934	0.023208544260	0.000605008673	0.023257985063	0.000555567870
10	0.005393020742	0.005248340743	0.000144679998	0.005259205713	0.000133815028
			at $t = 0.1$		
0	1.661469164844	1.655671296296	0.005797868547	1.655671296296	0.005797868547
2	1.071394686756	1.074295496614	0.002900809858	1.074298499509	0.002903812752
4	0.338632114578	0.338736397700	0.000104283121	0.338811112589	0.000178998010
6	0.083026200964	0.082835431504	0.000190769459	0.082844796101	0.000181404862
8	0.019069404618	0.019010724633	0.000058679984	0.019012555774	0.000056848844
10	0.004313817894	0.004299737397	0.000014080497	0.004300139804	0.000013678090

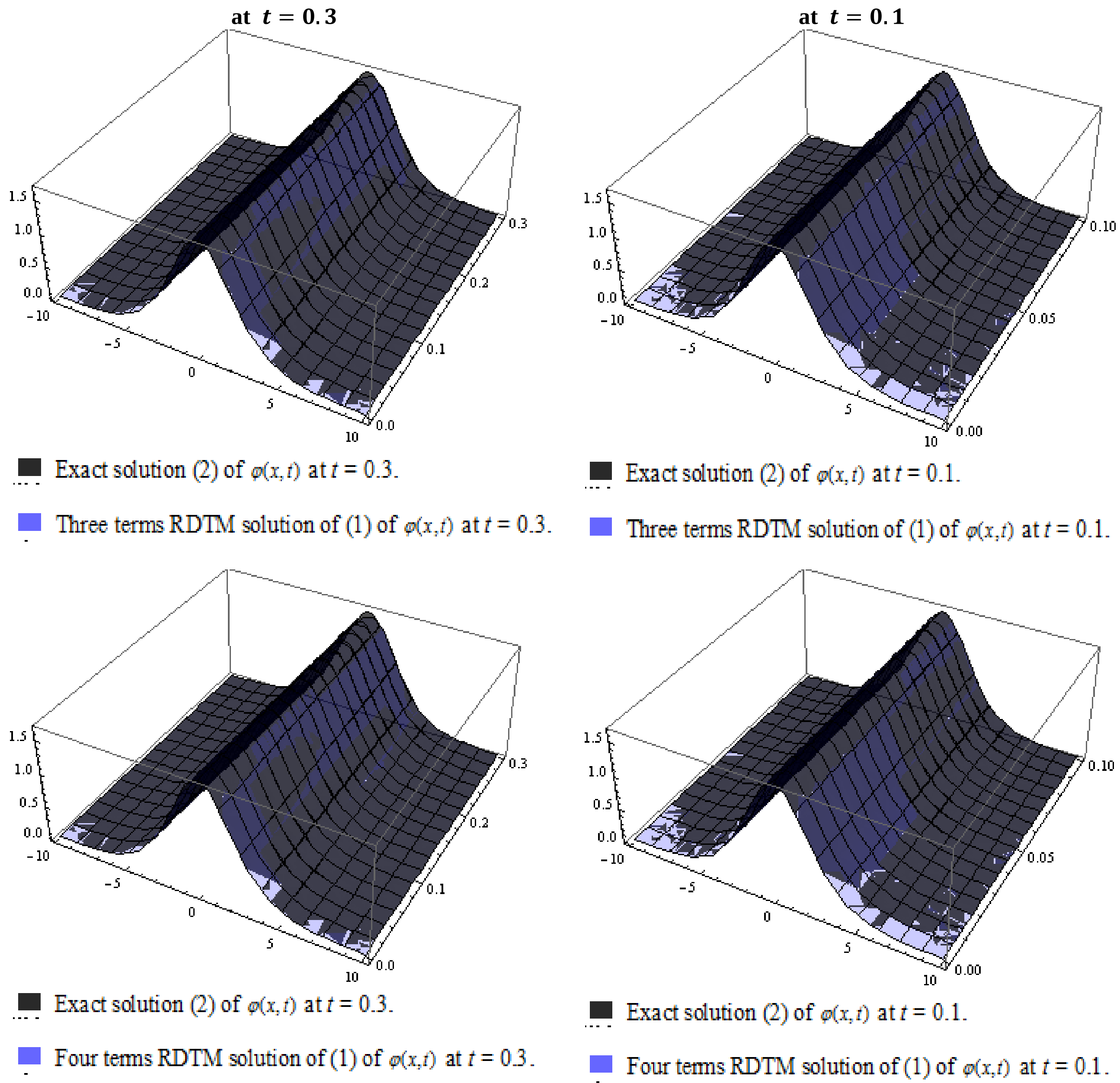

Fig 2. Comparison of RDTM solution of coupled problem of SRLWE (1) with the exact solution $\varphi(x,t)$ of (2) at $t = 0.3$ and $t = 0.1$.

Example 2: While considering the coupled problem of SRLWE (1) with the following initial conditions:

$$\left.\begin{aligned} z(x,0) = z_0(x) = \frac{5}{2}\sec h^2 \frac{\sqrt{5}}{6} x^2 \\ \varphi(x,0) = \varphi_0(x) = \frac{5}{3}\sec h^2 \frac{\sqrt{5}}{6} x^2 \end{aligned}\right\}. \qquad (12)$$

The reduced differential transformation for (12) was as under

$$\left.\begin{aligned} Z_0(x) = \frac{5}{2}\sec h^2 \frac{\sqrt{5}}{6} x^2 \\ \Phi_0(x) = \frac{5}{3}\sec h^2 \frac{\sqrt{5}}{6} x^2 \end{aligned}\right\}. \qquad (13)$$

The following tables have shown the comparison of the numerical results of the coupled problem of SRLWE (1) with the exact solution (2).

The comparison of RDTM solution of coupled problem of SRLWE (1) with the exact solution $z(x,t)$ of (2) at $t = 0.3$ and $t = 0.1$ is given as under

Table 4: Comparison of RDTM solution of coupled problem of SRLWE (1) with the exact solution $z(x,t)$ of (2) at $t = 0.3$ and $t = 0.1$.

		at $t = 0.3$			
x	**Exact Solution**	**Three Terms RDTM**	**Absolute Error**	**Four Terms RDTM**	**Absolute Error**
0	2.430985151677	2.500000000000	0.069014848322	2.500000000000	0.069014848322
2	1.821512232731	1.146462937137	0.675049295594	1.311466617106	0.510045615625
4	0.618480322493	0.000246566588	0.618233755904	0.000283388616	0.618196933876
6	0.154766415941	0.000000000140	0.154766415801	0.000000000185	0.154766415756
8	0.035720329401	0.000000000000	0.035720329401	0.000000000000	0.035720329401
10	0.008089531113	0.000000000000	0.008089531113	0.000000000000	0.008089531113
		at $t = 0.1$			
0	2.492203747266	2.500000000000	0.007796252733	2.500000000000	0.007796252733
2	1.607092030134	0.628658827564	0.978433202570	0.634770074970	0.972321955164
4	0.507948171867	0.000103763815	0.507844408052	0.000105127594	0.507843044273
6	0.124539301446	0.000000000044	0.124539301402	0.000000000045	0.124539301400
8	0.028604106928	0.000000000000	0.028604106928	0.000000000000	0.028604106928
10	0.006470726842	0.000000000000	0.006470726842	0.000000000000	0.006470726842

The comparison of RDTM solution of coupled problem of SRLWE (1) with the exact solution $\varphi(x,t)$ of (2) at $t = 0.3$ and $t = 0.1$ is given below

Table 5: Comparison of RDTM solution of coupled problem of SRLWE (1) with the exact solution $\varphi(x,t)$ of (2) $t = 0.3$ and $t = 0.1$.

		at $t = 0.3$			
x	**Exact Solution**	**Three Terms RDTM**	**Absolute Error**	**Four Terms RDTM**	**Absolute Error**
0	1.620656767784	1.666666666666	0.046009898881	1.666666666666	0.046009898881
2	1.214341488487	0.864978524097	0.349362964390	0.947400342329	0.266941146158
4	0.412320214995	0.000230150008	0.412090064987	0.000285353176	0.412034861819
6	0.103177610627	0.000000000126	0.103177610501	0.000000000194	0.103177610433
8	0.023813552934	0.000000000000	0.023813552934	0.000000000000	0.023813552934
10	0.005393020742	0.000000000000	0.005393020742	0.000000000000	0.005393020742
		at $t = 0.1$			
0	1.661469164844	1.666666666666	0.005197501822	1.666666666666	0.005197501822
2	1.071394686756	0.450856442626	0.620538244130	0.453909102560	0.617485584195
4	0.338632114578	0.000091098277	0.338541016301	0.000093142839	0.338538971739
6	0.083026200964	0.000000000040	0.083026200923	0.000000000042	0.083026200921
8	0.019069404618	0.000000000000	0.019069404618	0.000000000000	0.019069404618
10	0.004313817894	0.000000000000	0.004313817894	0.000000000000	0.004313817894

One can see in above Examples 1 and 2 that corresponding to $t = 0.3$, using four terms RDTM, when x was increased from 0 to 10, the solution converged faster to the actual solution as compared with three terms RDTM. The same was true for $t = 0.1$. Thus, the above tables and graphs demonstrated that, in general, when x was becoming larger and larger while t was made smaller and smaller, the convergence rate was becoming faster. Since, partial differential equations in the coupled problem under consideration had complicated mixed partial derivatives (Handibag and Karande, 2012). The obtained results have drawn attention to the fact that RDTM was an applicable technique for finding solution of coupled system of nonlinear partial differential equations. The study confirmed the computational stability and efficacy of the method as reported by (Al-Amr, 2014; Srivastava *et al*., 2014 and Yildirim, 2012).

It was noteworthy to mention that if readers wanted to compare results with boundary value problems then they could follow the following boundary condition reported by (Hu *et al*., 2014) as is given below

$$z(x_L,t) = z(x_R,t) = 0, \quad \varphi(x_L,t) = \varphi(x_R,t) = 0 \ , \quad t \in [0,T]. \qquad (14)$$

There were many numerical methods which were applied to estimate the solutions of the nonlinear PDEs and related problems as has been reported by (Tadmor, 2012 and Bartels, 2015). The prototype solutions were found by using the generalized kudryashov approach to SRLWE (Bulut *et al*., 2015). In another methodology, the cubic B-spline was applied for obtaining numerical results of SRLWE after discretizing the equation reported by (Mittal and Tripathi, 2015). The meshless kernel based technique was also applied for obtaining the solution of SRLWE (Dereli, 2016). Many problems have been solved by RDTM using linear and nonlinear equations without applying any complicated polynomials reported by (Saravanan and Magesh, 2013). It was worth mentioning that most of the problems dealt in the context of the reduced differential transform method did not deal with the coupled problem when there were mixed partial derivatives. The current study mainly dealt with such types of implications of partial differential equations.

In the above given references, it could be seen that the investigation of exact solutions of nonlinear partial differential equations played a significant role in the study of physical phenomena of nonlinear problems. A number of approaches including exact, purely numerical and approximate techniques are presented in the literature for obtaining the solution of nonlinear partial differential equations reported by (Khan *et al*., 2015; Lopez-Sandoval *et al*., 2015 and Naher, 2016).

This study was important for the coupled problem when nonlinearity and mixed partial derivatives occurred in the system of equations. This semi analytical technique was reliable and very much efficient, and converged quickly to the accurate solutions. The results obtained were of considerable interest. The complicated mathematical expressions were calculated by constructing various algorithms in Wolfram Mathematica 9.0 for ensuring the proficiency of the method. The derived numerical computations will be helpful for further research.

Conclusion: A coupled problem of symmetric regularized long wave equations was studied by applying the reduced differential transform method. The method was implemented in Wolfram Mathematica 9.0 and executed the program many times by taking various intervals of time and step sizes. The obtained numerical results were compared with the exact solution. It was concluded from the obtained numerical and graphical results that the reduced differential transform method turned to be very stable scheme.

REFERENCES

Abazari, R.and M. Abazari, (2013). Numerical study of Burgers–Huxley equations via reduced differential transform method. Comp. Appl. Math. 32: 1-17.

Al-Amr, M. O. (2014). New applications of reduced differential transform method. Alexandria Eng. J. 53(1): 243–247.

Ayaz, F. (2004). Solutions of the system of differential equations by differential transform method. Appl.Math.Comput. 147: 547–567.

Bartels, S. (2015). Numerical methods for nonlinear partial differential equations. Vol. 47. Springer Series in Computational Mathematics: London.

Bulut, H., Baskonus, H. M., and E. Cüvelek, (2015). On the prototype solutions of symmetric regularized long wave equation by generalized kudryashov method. Math. Lett. 1(2): 10-16.

Cenesiz, Y., Keskin, Y., and A. Kurnaz, (2010). The Solution of the nonlinear dispersive $K(m, n)$ equations by RDT method. Int. J. Nonlin. Sci. 9(4): 461-467.

Chen, C-K. and S-H. Ho, (1999). Solving partial differential equations by two-dimensional differential transform. Appl.Math.Comput. 106: 171–179.

Debnath, L. (1997). Nonlinear partial differential equations for scientists and engineers. 2nd Ed. Boston: Birkhauser. 151 p.

Dereli, Y. (2016). Solution of symmetric RLW equation by the meshless kernel based method of lines. CBU J. Sci. 12(2): 179-185 .

Handibag, S., and B-D. Karande, (2012). Laplace substitution method for solving partial differential equations involving mixed partial derivatives. Int. J. Pure Appl. Math. 78(7): 973-979.

Hu, J., Zheng, K., and M. Zheng, (2014). Numerical simulation and convergence analysis of a high-order conservative difference scheme for SRLW equation. Appl. Math. Model.38: 5573–5581.

Khan, K., Akbar, M-A., and H. Koppelaar, (2015). Study of coupled nonlinear partial differential equations for finding exact analytical solutions. R. Soc. Open Sci. 2(7): 140-406.

Keskin, Y. (2010 a). Application of reduced differential transformation method for solving gas dynamics equation. Int. J. Contemp. Math. Sci. 5(22): 1091-1096.

Keskin, Y., and G. Oturanc, (2010 b). Numerical solution of regularized long wave equation by reduced differential transform method. Appl. Math. Sci. 4(25): 1221-1231.

Keskin, Y., and G. Oturanc, (2010 c). Reduced differential transform method for generalized KDV equations. Math. Comput. Appl. 15(3): 382-393.

Keskin, Y., and G. Oturanc, (2010 d). Reduced differential transform method for solving linear and nonlinear wave equations. Iran. J. Sci. Technol. Transaction A. 34(A2).

Keskin, Y., and G. Oturang, (2009). Reduced differential transform method for partial differential equations. Int. J. Nonlin. Sci. Num. 6(10): 741-749.

Lopez-Sandoval, E., Mello, A., and J. J. Godina Nava, (2015). Series Solution of non linear partial differential equations from Mathematical Physics.*CoRR.5*:17. https://arxiv.org/abs/1206.2346

Mittal, R. C., and A. Tripathi, (2015). Numerical solutions of symmetric regularized long wave equations using collocation of cubic B-splines finite element. Int. J. Comput. Method. Engg. Sc. Mech. 16(2): 142-150.

Naher, H. (2016). Algorithms and some new traveling wave solutions via new extension of (G'/G) - expansion method of coupled nonlinear equations. British J. Math. Comp. Sci. 11(6): 1-13.

Rawashdeh, M. (2013). Using the reduced differential transform method to solve nonlinear PDEs arises in Biology and Physics. World Appl. Sci. J. 8(23): 1037-1043.

Saravanan, A., and N. Magesh, (2013). A comparison between the reduced differential transform method and the Adomian decomposition method for the Newell–Whitehead–Segel equation. Egypt. Math. Soc. 21: 259–265.

Seyler, C. E., and D-L.Fenstermacher, (1984). A symmetric regularized longwave equation. Phys. Fluids. 27(1): 4-7.

Srivastava, V. K., Mishra, N., Kumar, S., Singh, B. K., and M. K. Awasthi, (2014). Reduced differential transform method for solving (1+n)-dimensional Burgers' equation. Egyptian .J. B. App. Sc. 1(2): 115–119.

Tadmor, E. (2012). A Review of Numerical Methods for Nonlinear Partial Differential Equations. Bull. New Ser. Am. Math. Soc. 49(4): 507–554.

Tari, A., Rahimi, M-Y., Shahmorad, S., andF. Talati, (2009). Solving a class of two-dimensional linear and nonlinear Volterra integral equations by the differential transform method. J. Comput. Appl. Math. 228: 70-76.

Wang, T., Zhang, L., and F.Chen, (2007). Conservative schemes for the symmetric regularized long wave equations. Appl.Math.Comput. 190: 1063-1080.

Yildirim, K. (2012). A solution method for solving systems of nonlinear PDEs. World Appl. Sci. J. 11(18): 1527-1532.

Zhou, J. (1986). Differential transformation and its application for electrical circuits(in Chinese). Wuuhahn , China: Huarjung University Press.