INTEGRATED APPROACH OF SET THEORY AND PATTERN SEARCH METHODS FOR SOLVING AIRCRAFT ATTACKING PROBLEM

M. F. Tabassum, M. Saeed, N. A. Chaudary*, J. Ali and A. Sana

Department of Mathematics, University of Management and Technology, Lahore.
*Department of Mathematics, UCEST, Lahore Leads University, Lahore.
Corresponding Author: farhanuet12@gmail.com

ABSTRACT: The formulation and solution of aircraft attacking problem, having supply of fuel and aircrafts availability constraints was determined. Soft\fuzzy set theory and statistical approaches were used for air craft selection and probability of destruction respectively. The pattern search methods were used for the optimization of aircraft attacking problem. These methods were essentially intended for unconstrained optimization problems. In formulated aircraft attacking problem the constraints were controlled by using exterior penalty functions. MATLAB environment was used to calculate the results of the aircraft attacking problem, which exhibited the efficiency of methods. Comparative study was also provided for the best selection of these pattern search methods. Finally it was concluded that the probability of failure was *0.0000013* to demolish the targets. Numerical results witnessed the remarkable low computational cost behavior of Hooke and Jeeve,s method which was the best one.

Keywords: Pattern search methods, aircraft attacking problem, penalty function and unconstrained optimization.

(Received 08-06-2016 Accepted 12-01-2017)

INTRODUCTION

Pattern search methods (PSM's) persist for a number of good quality reasons. PSM's work excellently in practice and have been beneficial when other elegant approaches are unsuccessful. PSM's are the methods of first recourse, knowledgeable users. PSM's are convincingly forthright to apply and can be functional practically for many non-linear problems related to optimization (Rardin, 2002 and Rao, 2002).

PSM's are also called direct search methods which transform the objective function into number of finite points for iteration process, and also making decision for dealing to take next exclusively depended on the function values (Marazzi and Nocedal, 2002). PSM's are constructed on operations for simplexes like shrink, contractions, reflections or expansions. The sampling of these PSM's is directed by sets of directions with appropriate features (Moré and Wild, 2007).

It is easy to demonstrate maximum positive bases in every simplex which having $n + 1$ vertices. Commonly, when positive basis are provided, it is straight forward to isolate simplexes in which the number of vertices are $n + 1$. The difficulty which is now under consideration is to transform the constrained problem to an unconstrained real-valued optimization problem. Such kind of problem is also stated in present study. The first time simplex based operations are introduced for optimization purpose in the work carried out by (Nelder and Mead, 1965). In that approach, the worst vertex of a simplex is reflected through centroid and generates the new point. It tried to improve the simplex with respect to other n vertices (Mckinnon, 1998 and Price *et al.*, 2002).

Pattern search methods for unconstrained optimization have also been generating the idea of the multi-directional search algorithm. PSM's are embraced by the fact that results produced on the information of function value (Lewis *et al.*, 2000). PSM's like Hooks-Jeeves and Nelder-Mead have been employed before multi-directional search algorithm. Most of the pattern search methods are simple to use and uncomplicated. Two basic significant points are involved in multi-directional search method; first consideration of decreasing direction in function values with appropriate length at each iterate, secondly the algorithm must find out appropriate fitting step length (Kolda *et al.*, 2003).

Most of the methods are designed based on penalty functions, to tackle constraints (Rardin, 2002). Penalty functions and barrier functions were initially invented in 1940s and extended by Carroll, McCormick and Fiacco in the 1960s. The basic idea of these methods is to change, modify or convert the constrained optimization problem into an unconstrained optimization problem by adding or subtracting a penalty value to or from the objective (Ashok and Chandrugupta, 2011).

In this paper the propose strategy is categorized as: Firstly the air crafts have selected by using the soft and fuzzy set theory. Secondly the features of PSM's have been discussed that differentiate the methods from other methodologies of nonlinear optimization. Thirdly the model of aircraft attacking problem has been formulated and converted it from constrained to unconstrained optimization use penalty function. Finally, the unconstrained optimization problem has been implemented on these PSM's and compared the results.

MATERIALS AND METHODS

The pattern search methods were used for optimizing the aircraft attacking problem with constraints. In formulated optimization aircraft attacking problem the constraints were tackled by using exterior penalty functions. By selecting the best method there was a necessity to conduct comparative studies of their potential applications to modern world problems, like formulated in the present study.

Hooke-Jeeves Method: For an N-dimensional problems, Hooke-Jeeve's (HJ) method required an initial point x_0, a number of N independent search directions v_i, step-length parameters $\delta_i > 0$ and a parameter $\mu > 1$ (Price *et al.*, 2002). The method used two types of moves given below:

Exploratory Move: This move was made on the current point by investigation along each direction according to the following formula:

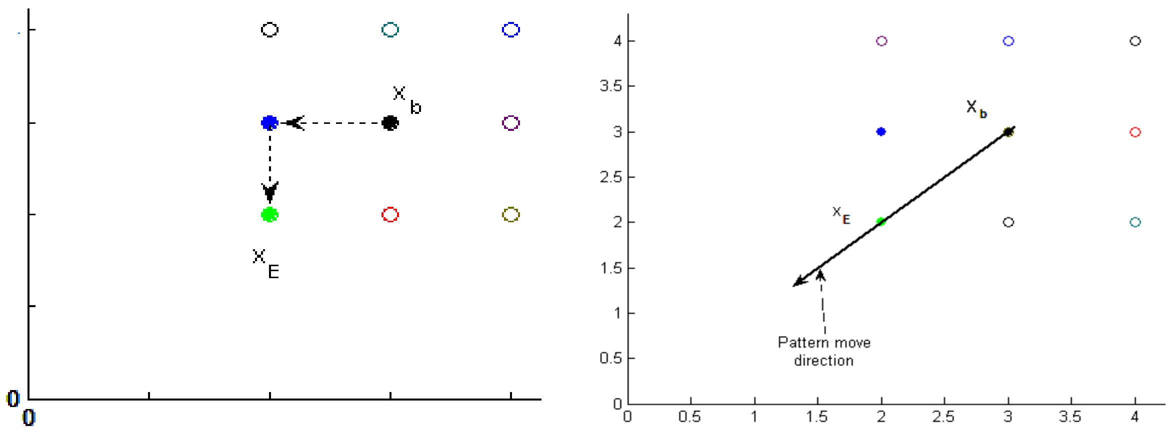
$$x_{new} = x_0 \pm \delta_i v_i \quad for\ all\ i = 1, 2, 3, \ldots, N.$$

Pattern Move: After the completion of successfully exploratory move execution of pattern move was made, by shifting present base point with concerning direction and got a new point. When a pattern move was completed it was promising to move in that direction as much as allowed. The parameter η, used for enlargement $\eta \geq 1$, was used for this pattern move (Tabassum *et al.*, 2015). The pattern direction was calculated by the help of this formula:

$$d = z_E - z_b.$$

A new point through pattern move was found as given below (Hooke and Jeeves, 1961):

$$y_b = z_E + \eta d = z_E + \eta (z_E - z_b).$$



(a) (b)

Fig-1. (a) Successful exploratory move (b) Pattern move direction

Nelder-Mead Simplex Method: In 2-dimensions, Nelder-Mead simplex (NM) method started with an initial simplex of vertices, y^2 = Worst Point, y^1 = Good Point, y^0 = Best Point.

Centroid point y^C was the midpoint of best and good vertices, worst point reflected through centroid and the new point y^r created at equal distance from centroid to worst point. In this method there were several operations to be performed (Tabassum, 2015). Reflection occurred when $y^1 \geq y^r > y^0$ which produced the reflected point y^r obeying $y^r = y^C + \delta^R(y^c - y^n)$. The expansion took place when $y^1 \geq y^0 > y^e$ and resulted in the expanded point y^e satisfying the relation $y^e = y^C + \delta^e(y^c - y^n)$. If the reflected point was located between good and the best points then a contraction took place. Outside type contraction occurred when $y^2 \geq y^r > y^1$.

Mathematically, the outside contracted point y^{OC} was given as

$$y^{OC} = y^C + \delta^{OC}(y^c - y^n)$$

Inside type contraction occurred when $y^r \geq y^2$. Mathematically, the point y^{iC} was given as

$$y^{IC} = y^C + \delta^{iC}(y^c - y^n)$$

If all above conditions failed then shrink was applied in simplex.

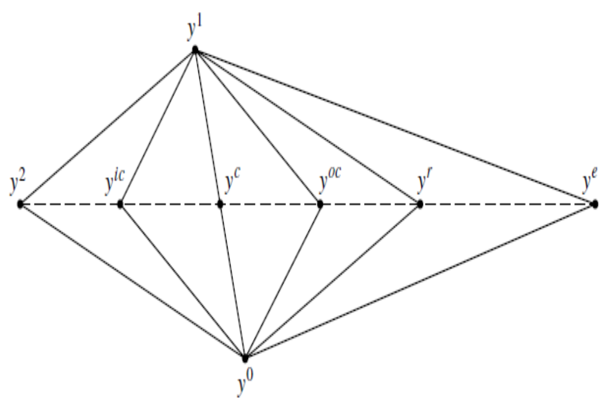


Fig-2. Steps of Nelder–Mead method.

Multi-directional Search method: Multi-directional search (MDS) method also initialized like the Nelder-Mead search method by taking *N + 1* point in N independent search directions (Sana, 2016). Mathematically, the reflected points y^{RW} *and* y^{RG} were calculated by using the following relation

$$min\{f(y_i^k), i = 1, \dots, n\} < f(y_0^k)$$

Mathematically, the expanded points y^{EW} *and* y^{EG} were calculated by using the following relation

$$min\{f(y_{ei}^k): i = 1, \dots, n\} < min\{f(y_{ri}^k), \quad i = 1, \dots, n\}$$

Mathematically, the inside contraction of points y^{CW} *and* y^{CG} were calculated by using the following relation

$$min\{f(y_{ci}^k), i = 1, \dots, n\} < f(y_0^k)$$

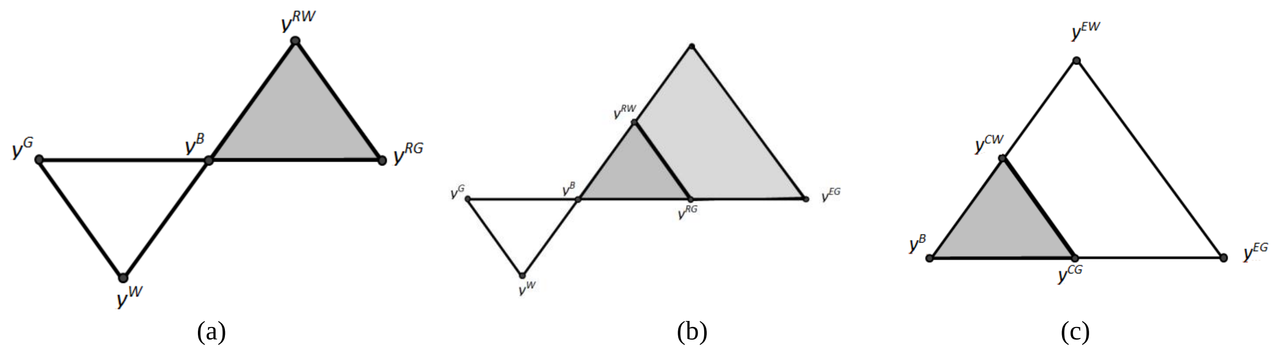


Fig-3. (a) Reflection (b) Expansion (c) Contraction in multi-directional search method

Aircraft attacking problem: Air force received orders to eliminate the enemy four targets. Four key targets (nuclear plants) located at different cities. For this particular mission the limited fuel supply 40,000 liters was available. Any aircraft sent to any city which has extra 100 liters fuel for safety purpose. Armament details of air crafts were available in net source (NS1 and NS2, 2016).

Table-1. Specific details of available bomber aircraft.

Air Crafts	Speed	Range	Availability of Air Crafts	Fuel Consumption Air Crafts	Dimensions	Weight (Empty)
Chengdu F-7/J7 Air guard	1350 mph	600 km	24	4 L/km	L=13.945m, H=4.103m	5275 kg
Chengdu J-10	1408 mph	1850 km	20	3 L/km	L = 14.5 m, H = 6 m	6940 kg
Chengdu FJ-17 Thunder	1218 mph	1200 km	15	2.5L/km	L = 14 m, H = 5.1 m	6320 kg
Mirage III	863 mph	685 km	18	2L/km	L = 15.5 m, H = 4.5 m	6600 kg
Mirage V	1188 mph	4000 km	22	2L/km	L=15.55 m, H = 4.5 m	6600 kg
Martin F-16A/B Falcon	1320 mph	3886 km	15	3L/km	L=15.03 m, H = 5.1 m	8273 kg
Martin F-16 C/D Falcon	1320 mph	3886 km	20	2.5 L/km	L=15.03 m, H = 5.09 m	8273 kg
Q-5 Fantan	739 mph	2000 km	9	4L/km	L=15.65 m, H = 4.33 m	6375 kg

Table-2 Information about target and their probability of hitting by bomber aircraft.

Plants	Cities	Distance	Probability of Destruction		
			F	S	M
1	City-W	880	0.15	0.10	0.14
2	City-X	750	0.30	0.15	0.18
3	City-Y	920	0.25	0.12	0.16
4	City-Z	815	0.20	0.21	0.23

The objective was to minimize the probability of failure of the mission and to find how many aircraft (each type) were to be allocated across the four targets (Ruhul and Newton, 2008)?

Selection of Air Crafts: Fuzzy Sets: A fuzzy set *A* in universal set *U* in the form of order pairs is defined as
$A=\{(x , \mu_A(x)) : x \varepsilon U\}$
where $\mu_A \rightarrow [0,1]$ = mapping of all fuzzy and $\mu_A(x)$ of I.
I^U = the family of all fuzzy sets in *U* (Onyeozili, 2013).
The *α*-level set of a fuzzy set *F* was defined by:
$F(\alpha)=\{x \varepsilon U : \mu_F(x) \geq : \alpha \}$, where $\alpha \varepsilon [0,1]$.
It was supposed $U = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}$

and single parameter quality of air crafts which were characterized by the value set whose terms were {best, good, average, and poor}. Let the terms best for example be associated with its own fuzzy set as follows:
$F_{best} = \{(P_2, 0.2), (P_5, 0.6), (P_6, 0.9), (P_7, 1.0)\}$
F_{best} (0.5)= $\{P_5, P_6, P_7\}$, F_{best} (0.7)= $\{P_6, P_7\}$, F_{best} (0.9)= $\{P_6, P_7\}$, F_{best} (1.0)= $\{P_7\}$
Where A = {0.2, 0.6, 0.9, 1.0} ⊂ [0, 1] which can be regarded as the parameter set such that $F_{best}: A \longrightarrow P(U)$ gave the approximate value set $F_{best}(\propto)$, for $\propto \in A$. Thus, the soft set for the fuzzy set F_{best} could be written as;
$(F_{best}, A)= \{(0.2, \{P_2, P_5, P_6, P_7\}), (0.5, \{P_5, P_6, P_7\}), (0.7, \{P_6, P_7\}), (0.9, \{P_6, P_7\}), (1.0, \{P_7\})\}$

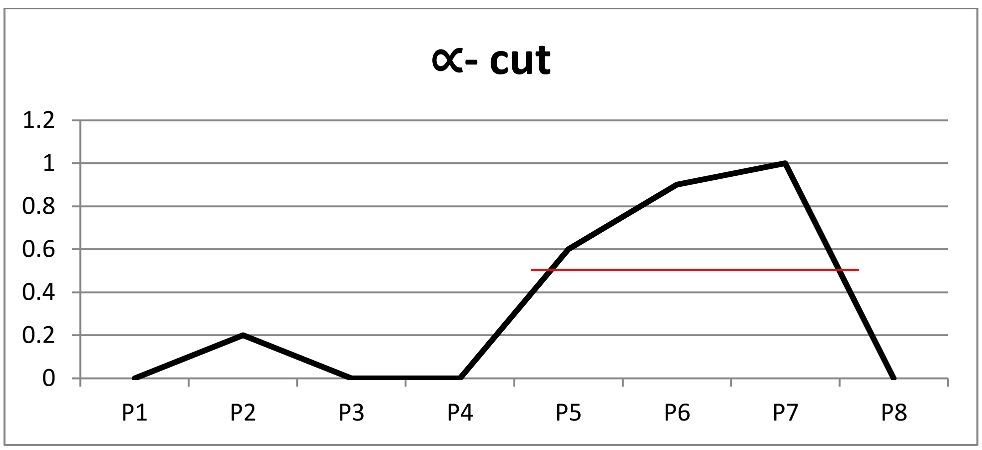


Fig-4. Representation of ∝–cut at 0.5

Soft Sets: Let U = initial universe set, E = set of parameters and P(U) = power set of U and A contain in E. A pair (F, E) was called a soft set if and only F : E → P(U) (Maji, 2003).
U = set of air crafts available for mission.
E = set of parameters (word or a sentence) = {maximum range; maximum speed; air to air missile facility; cannon with 500 rounds; air to ground bombardment; Nuclear Weapon capacity}.

In this particular case a soft set was meant to focus the capabilities of air crafts. The soft set *(F, E)* described the "attractiveness of air crafts" which Pakistani Air Force was going to select. There were eight air crafts in the universe *U* given by

$$U = \{ P_l, P_2, P_3, P_4, P_5, P_6, P_7, P_8 \}$$

$$E= \{ e_l, e_2, e_3, e_4, e_5, e_6)$$

Where e_l = maximum speed 1000 mph
e_2 = maximum range 2000 km
e_3 = air to air missile facility
e_4 = cannon with 500 rounds
e_5 = air to ground bombardment
e_6 = Nuclear Weapon capacity

It was supposed
$F(e_1) = \{P_2, P_3, P_5, P_6, P_7\}$ $F(e_2) =\{P_5, P_6, P_7, P_8)$
$F(e_3) = \{P_2, P_5, P_6, P_7\}$ $F(e_4) =\{P_4, P_5, P_6, P_7)$
$F(e_5) =\{P_1, P_2, P_4, P_7, P_8)$ $F(e_6) = \{P_1, P_5, P_8\}$

The soft set (F, E) was a parameterized family $\{F(e_i), i = 1,2,3,\dots ,6 \}$ of subsets of the set U and it provided a group of approximate detail of objects.

Table-3 Tabular Representation of air craft in soft set theory.

U	e_1	e_2	e_3	e_4	e_5	e_6	Choice Value
P_1	0	0	0	0	1	1	2
P_2	1	0	1	0	1	0	3
P_3	1	0	0	0	0	0	1
P_4	0	0	0	1	1	0	2
P_5	1	1	1	1	0	1	**5**
P_6	1	1	1	1	0	0	**4**
P_7	1	1	1	1	1	0	**5**
P_8	0	1	0	0	1	1	3

Table-4. Selected Air Crafts through Fuzzy and Soft set theory.

Bomber Type	Description	Km/L	Availability
F	Mirage V	2	22
S	Martin F-16 A/B Falcon	3	15
M	Martin F-16 C/D Falcon	2.5	20

Formulation of aircraft attacking problem

Decision Variables: z_{ij} = air craft types i (i may be F-type, S-type and M-type) that were sent targets j (j =1, 2, 3 and 4).

Objective Function: The objective was to minimize the probability of failure in destroying the targets.

Probability of success to destroy target 1 by F aircraft = 0.15.

Probability of failure to destroy target 1 by one F aircraft = 1 - 0.15 = 0.85

Probability of failure to destroy target 1 by z_1 number of F aircraft = $(0.85)^{z1}$

Similarly the probability of failure to destroy target 2 by z_2 number of F aircraft = $(0.70)^{z2}$

Probability of failure to destroy target 3 by z_3 number of F aircraft = $(0.75)^{z3}$

Probability of failure to destroy target 4 by z_4 number of F aircraft = $(0.80)^{z4}$

In same manners the other probabilities were calculated.

Probability of failure to destroy target 1 by z_5 number of S aircraft = $(0.90)^{z5}$

Probability of failure to destroy target 2 by z_6 number of S aircraft = $(0.85)^{z6}$

Probability of failure to destroy target 3 by z_7 number of S aircraft = $(0.88)^{z7}$

Probability of failure to destroy target 4 by z_8 number of S aircraft = $(0.79)^{z8}$

And also

Probability of failure to destroy target 1 by z_9 number of M aircraft = $(0.86)^{z9}$

Probability of failure to destroy target 2 by z_{10} number of M aircraft = $(0.82)^{z10}$

Probability of failure to destroy target 3 by z_{11} number of M aircraft = $(0.84)^{z11}$

Probability of failure to destroy target 4 by z_{12} number of M aircraft = $(0.77)^{z12}$

So the objective function was

Minimize $Z = (0.85)^{z1} \times (0.70)^{z2} \times (0.75)^{z3} \times (0.80)^{z4} \times (0.90)^{z5} \times (0.85)^{z6} \times (0.88)^{z7} \times (0.79)^{z8} \times (0.86)^{z9} \times (0.82)^{z10} \times (0.84)^{z11} \times (0.77)^{z12}$

Constraints

Fuel Supply Limitation (F Air Craft):

Fuel required (liters) up to target for each trip of z_1 for one side $= \frac{880}{2}$

Fuel required (liters)up to target for each trip of z_1 for comeback

$$= \left(\frac{880}{2}\right) \cdot 2$$

Total Fuel required including safty purpose for z_1

$$= \left(\frac{880}{2}\right) \cdot 2 + 100 = 980$$

Similarly

Total Fuel required including safty purpose for z_2 $= 850$

Total Fuel required including safty purpose for z_3 $= 1020$

Total Fuel required including safty purpose for z_4 $= 915$

Fuel Supply Limitation (S Air Craft):

Total Fuel required including safty purpose for z_5 $= 687$

Total Fuel required including safty purpose for z_6 $= 600$

Total Fuel required including safty purpose for z_7 $= 714$

Total Fuel required including safty purpose for z_8 $= 644$

Fuel Supply Limitation (M Air Craft):

Total Fuel required including safty purpose for z_9 $= 804$

Total Fuel required including safty purpose for z_{10} $= 700$

Total Fuel required including safty purpose for z_{11} $= 836$

Total Fuel required including safty purpose for z_{12} $= 752$

The fuel supply constraint is

$$980\,z_1 + 850z_2 + 1020z_3 + 915z_4 + 687z_5 + 600z_6 + 714\,z_7 + 644z_8 + 804z_9 + 700z_{10} + 836z_{11} + 752z_{12} \le 40{,}000$$

Constraint for the number of aircraft:

Availability of total F-type aircrafts: $z_1 + z_2 + z_3 + z_4 \le 22$

Availability of total S-type aircrafts: $z_5 + z_6 + z_7 + z_8 \le 15$

Availability of total M-type aircrafts: $z_9 + z_{10} + z_{11} + z_{12} \le 20$

Finally the non-linear aircraft attacking optimization problem

Minimize $Z = (0.85)^{z1} \times (0.70)^{z2} \times (0.75)^{z3} \times (0.80)^{z4} \times (0.90)^{z5} \times (0.85)^{z6} \times (0.88)^{z7} \times (0.79)^{z8} \times (0.86)^{z9} \times (0.82)^{z10} \times (0.84)^{z11} \times (0.77)^{z12}$

Subject to

$$980\,z_1 + 850z_2 + 1020z_3 + 915z_4 + 687z_5 + 600z_6 + 714\,z_7 + 644z_8 + 804z_9 + 700z_{10} + 836z_{11} + 752z_{12} \le 40{,}000$$

$$z_1 + z_2 + z_3 + z_4 \le 22$$

$$z_5 + z_6 + z_7 + z_8 \le 15$$

$$z_9 + z_{10} + z_{11} + z_{12} \le 20$$

$$z_{ij} \ge 0 \qquad \forall\, i \,\&\, j$$

The exterior penalty function had been used to convert the constrained optimization problem into unconstrained optimization problem as following (Ravindran, 2006; Ruhul, 2008; Ashok, 2011).

$$P(x) = r(\max[0, g_1(x), g_2(x), g_3(x)])$$

On the other hand, the class of interior penalty functions involving log penalties and barrier functions had been designed to work when an algorithm implemented the simulations started from a feasible initial guess. But finding a feasible initial guess under the inequality constraints had been a great challenge in the optimization process. Therefore such a drawback restricted the current study to use exterior penalty function approach for handling constraints.

Minimize $Z = (0.85)^{z1} \times (0.70)^{z2} \times (0.75)^{z3} \times (0.80)^{z4} \times (0.90)^{z5} \times (0.85)^{z6} \times (0.88)^{z7} \times (0.79)^{z8} \times (0.86)^{z9} \times (0.82)^{z10} \times (0.84)^{z11} \times (0.77)^{z12} + 100[\max(0,[980\,z_1 + 850z_2 + 1020z_3 + 915z_4 + 687z_5 + 600z_6 + 714\,z_7 + 644z_8 + 804z_9 + 700z_{10} + 836z_{11} + 752z_{12} \le 40{,}000],[z_1 + z_2 + z_3 + z_4 - 22],[z_5 + z_6 + z_7 + z_8 - 15],[z_9 + z_{10} + z_{11} + z_{12} - 20])]$

Table-5. Parameters for pattern search methods.

Hooks and Jeeves Method	Nelder and Mead Method	Multi-Directional Search Method
Step length	Reflection coefficient	Expansion coefficient
$\Delta = (0.5, 0.5)^t$	$\delta_r = 1$	$\mu = 2$
Reduction parameter	Expansion coefficient	Contraction coefficient
$\alpha = 2$	$\delta_e = 2$	$\theta = 0.5$
	Inner-contraction coefficient	
	$\delta_{ic} = -05$	
	Outer-contraction coefficient	
	$\delta_{oc} = 0.5$	

RESULTS AND DISCUSSION

Pattern search methods were known because of their reliability, simplicity and flexibility in the field of operations research and optimization (Lewis *et al,* 2000). Convergence of stationary point(s) had been shown to satisfy the minimizer conditions. It seemed extraordinary that the given PSM's not required derivative information. In most of the PSM's to investigate the behavior of functions, spanning directions in search space were sufficient (Nelder and Mead, 1965).

As per study conducted by (Tabassum, 2015) reported the solution of the above formulated problem with different setting of parameters. The initial guess was taken as (0, 2, 2, 2, 2, 2, 2) for Nelder-Mead method, 57 iterations had been performed and the optimum value was obtained which was 0.00039273 after 105 function evaluation. The optimal point for the given value was (6.9058, 7.8245, 5.0665, 3.1737, 3.3790, 4.2369). Similarly the initial guess has been taken as (1, 2, 0, 1, 2, 3) for Hooke-Jeeves method, 13 iterations were performed and the optimum value was obtained which was 0.00088 after 165 function evaluation. The optimal points for the given values were (7, 8, 5, 3, 4, 5). The aircraft attacking problem was solved by using derivative free methods considering smaller values of the parameters. The best solution of the problem also gave the probability of failure destroying all targets by bomber aircrafts was 0.00039273 and 0.00088 (Ruhul and Newton, 2008).

The aircraft attacking problem was executed several times by taking the initial guess in the range of 1 to 10. The same value which was 1.143483E-5 at each initial guess was found which showed the consistent performance of these pattern search methods. The best results had been found by using Hooke-Jeeve's method in aircraft attacking problem, optimal point was (0, 7, 2, 9, 4, 1, 3, 7, 4, 5, 5, 2) provided the optimal value of 0.0000013 at 6 iterations. There were no constraints activated in the provided solution, so that the solution was feasible.

The second best solution was found by using Multidirectional search method in aircraft attacking problem, optimal points were (1, 8, 3, 8, 1, 1, 3, 9, 4, 5, 5, 2) provided the optimal value 0.00000918 at 5 iterations. The last solution had been found by using Nelder-Mead method in aircraft attacking problem, optimal points were (0, 7, 1, 6, 3, 1, 3, 11, 4, 4, 5, 2) providing the optimal value of 0.001095 at 47 iterations. The comparisons of the solutions found in this study are presented in table-6.

Table-6 Compression of all three PSM's.

	Methods		
	Nelder and Mead Method	**MDS Method**	**Hooke's and Jeeves Method**
Initial Guess	*1,8,3,8,4,1,3,9,4,5,5,2*	*1,8,3,8,4,1,3,9,4,5,5,2*	*1,8,3,8,4,1,3,9,4,5,5,2*
Function Value	1.143483E-5	1.143483E-5	1.143483E-5
Function Evaluations	*73*	*132*	7
Optimal Point	*0,7,1,6,3,1,3,11,4,4,5,2*	*1,8,3,8,1,1,3,9,4,5,5,2*	*0,7,2,9,4,1,3,7,4,5,5,2*
Function Value	0.0010952	0.000009918	0.0000013

The previous studies witnessed that NM method was comparatively a low computation cost method. On the other hand HJ method provided guaranteed convergence for a number of differentiable functions (Dimitri *et al*, 2000). The present study showed a different picture of the methods. It was observed that the solutions (Table 6) found by Hooke-Jeeve's method were the best. These comparisons witnessed that the deterministic PSM's like HJ, MDS and NM methods were yet better choices for solving such exponential type optimization problems in engineering design but HJ method was more reliable. And the final solution which was obtained was approximately 27 percent better than the solution provided in previous studies.

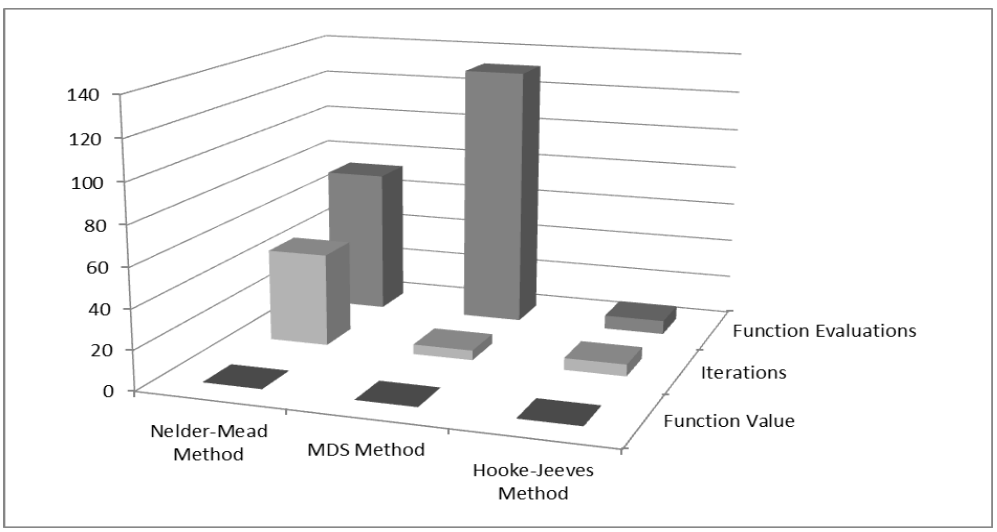


Fig-5. Pictorial representation of all three PSM's

The figure 5 shows the function values, iterations and function evaluations of all the three methods. HJ method terminated when the step length fell below 10^{-9} and NM terminated when the maximum of 200×No of variables function evaluations were carried out. At these termination criteria the function evaluations by HJ method were smaller than those of other methods. It was concluded that on the radical objective functions like the modeled one, HJ method may be a better with low cast choice.

For optimum results of aircraft attacking problem, a general-purpose solver was required. For numerical simulation of the aircraft attacking problem, the programming environment of MATLAB was found quite supportive due to availability of a plenty of built-in functions. Another important advantage of MATLAB was the fact that parameters were easily settled for handling constraints.

Finally the symmetries of air crafts were as fallows with probability of failure in destroying all targets. These results also satisfied the constraints of the given problem.

Mirage V bomber aircrafts were sent as (dispatched) =*0+7+2+9=18*

Martin F-16 A/B Falcon bomber aircrafts were sent as (dispatched) =*4+1+3+7*=15

Martin F-16 C/D Falcon bomber aircrafts were sent as (dispatched) =*4+5+5+2*=16

Probability of failure destroying all targets = 0.0000013

Conclusion: This research work reflected comparative study of three pattern search methods with the goal of identifying the most capable method for solving game theory optimization problems. In present study aircraft attacking problem was formulated and solved along with the fuel and availability of aircraft constraints. The novel techniques related to fuzzy and soft set theory were used for the selection of aircrafts. The exterior penalty function was used to convert the constrained optimization problem into unconstrained optimization problem which was the requirement of these pattern search methods. The statistics in table 6 and the convergence tracks depicted in figure 8 witnessed the superiority and efficiency of the Hooke-Jeene's method over the other two methods. Also the optimal value found by Hooke-Jeene's method was better than all of those values cited in literature. Finally it was concluded that Hooke-Jeene's method was better and effective option to solve game theory optimization problems.

REFERENCES

Ashok, D. B. and T. R. Chandrugupta, (2011). Optimization Concepts and Applications in Engineering. *Cambridge University Press*. (New York) 245 p

Dimitri, P. N. Bertsekas and J. N. Tsitsiklis, (2000). Gradient Convergence in Gradient Methods with Errors. *Siam J. Optim.* 10(3): 627–642

Hooke, R. and T. A. Jeeves (1961). Direct search Solution of numerical and statistical problems. *J. of the Assoc. for Comp. Machinery*. 8: 212-229

Kolda, T. G., R. M. Lewis and V. Torczon, (2003). Optimization by Direct Search Methods: New Perspective on Some Classical and Modern Methods. *Society for Indus. and Advanced Math.*, SIAM. 45(3):385-482

Lewis, R. M., V. Torczon and M. W. Trosset, (2000). Direct Search Methods: Then and Now. *J. of Comp. and App. Math*. 124: 191-207

Marazzi, M. and J. Nocedal, (2002). Wedge Trust region methods for derivative free optimization. *Math. Program*. 91: 289-305

Maji, P. K. (2003). Soft Set Theory. *Comp. and Math. with Appl.*, 45: 555-562

Mckinnon, K. I. (1998). Convergence of the Nelder–Mead simplex method to a non-stationary point. *SIAM J. Optim.* 9: 148–158

Moré, J. J. and S. M. Wild, (2007). Benchmarking derivative-free optimization algorithms. Tech. Report ANL/MCS-P1471-1207. *Math. and Comp. Sci. Division*, Argonne National Laboratory, Argonne.

Nelder, J. A. and R. Mead, (1965). A simplex method for function minimization. *Comput. J*. 7: 308–313

NS1: http://en.wikipedia.org/wiki/List_of_aircraft_of_the_Pakistan_Air_Force

NS2: http://www.combataircraft.com/en/Military-Aircraft/Pakistan/

Onyeozili, I. A., (2013). A Study of the Concept of Soft Set Theory and Survey of its Literature. *ARPN J. of Sci. and Tech*. 3(1): 61-68

Price, C. J., I. D. Coope, and D. Byatt, (2002). A convergent variant of the Nelder- Mead algorithm. *J. Optim. Theory Appl.* 113: 5–19

Rardin, R. L. (2002). Optimization in Operational Research. Pearson Education Press. (Singapore) 115 p

Ravindran A., K. M. Ragsdell and G. V. Reklaitis (2006). Engineering Optimization Methods and Applications. *John Wiley & Sons*, Inc. (Canada) 16, 139 p

Rao, S. S. (2002). Engineering Optimization Theory and Practice. John Wiley & Sons. (Canada) 98 p

Ruhul, A. S. and C. S. Newton, (2008). Optimization Modeling A Practical Approach. *CRC Press*. (USA)

Sana, A., M. Saeed, N. A. Chaudhry, M. F. Tabassum and M. Rafiq, (2016). Optimal Design of 16 bar Truss Structure by Pattern Search Methods. *Pak. J. of Sci.* 68(1): 371-376

Tabassum, M. F., M. Saeed, N. A. Chaudhry, A. Sana and Z. Zafar, (2015). Optimal design of oxygen production system by pattren search methods. *Pak. J. of Sci.* 67(4): 371-376

Tabassum, M. F., M. Saeed, N. A. Chaudhry, A. Sana and Z. Zafar, (2015). Optimal design of oxygen production system by pattren search methods. *Pak. J. of Sci.* 67(4): 371-376

Tabassum, M. F., M. Saeed , Nazir Ahmad, A. Sana, (2015). Solution of War Planning Problem Using Derivative Free Methods. *Sci. Int*. 27(1): 395-398

Tabassum, M. F., M. Saeed, A. Sana, and Nazir Ahmad, (2015). Solution of 7 Bar Tress Model Using Derivative Free Methods. *Proceedings of the Pak. Acad. of Sci.* 52 (3): 263–269.