RADIAL ARTIFICIAL BEE COLONY ALGORITHM FOR CONSTRAINT ENGINEERING PROBLEMS

M. Luqman, M. Saeed, J. Ali and M. F. Tabassum

Department of Mathematics, University of Management and Technology, Lahore, Pakistan.
Corresponding author's email: farhanuet12@gmail.com

ABSTRACT: A modified variant of artificial bee colony (ABC) algorithms, called radial artificial bee colony (RABC) algorithm, was proposed. This modified method incorporated two novel strategies during the initialization of employed bees and the determination of new locations for scout bees. RABC was applied to solve well-known constrained and unconstrained problems. The statistical results of RABC were compared with those of the original ABC and a number of approaches from the past studies. The comparisons revealed that RABC was superior to its competitor in terms of accuracy, speed of convergence and consistency.

Keywords: Radial artificial bee colony algorithm, Unconstraint optimization, Constrained optimization, Engineering design problems.

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INTRODUCTION

In most of the applications, in general, an optimization problem is described as

$$\textit{Minimize}\ \ f_\ell(\underline{x}); \quad \underline{x} \in \Re^d \ \textit{and} \quad 1 \le \ell \le M$$

Such that

$$h_i(\underline{x}) = 0, \quad 1 \le i \le p$$

$$g_j(\underline{x}) \le 0, \quad 1 \le j \le q$$

Where f_ℓ, h_i and g_j are real valued functions defined on $\Re^d$ (Yang and Gandomi, 2012).

In this problem, $x_1, x_2, x_3, ..., x_d$ are called design variables, the functions f_ℓ are considered as cost functions or objective functions, g_j and f_i are known as problem constraints and the space that is spanned by the design variables is known as search space or design space (Yang and Gandomi, 2012). The objective function can also be formulated as a maximization problem and the inequalities can also be expressed as greater than or equal to form (Tabassum *et al*., 2015, 2016).

Since the inclusion of algorithms like simulated annealing algorithm (SAA) and genetic algorithm (GA), the adaptations of diverse natural phenomena is an active and effective source of developing potential and efficient optimization techniques (Goldberg, 1989; Kirkpatrick *et al*., 1983). Particle swarm optimization (PSO) algorithm is based on the concept of swarm behaviors (Kennedy and Eberhart, 1995). Some examples of PSO-type algorithms are bats algorithm (BA), firefly algorithm (FA), artificial bee colony (ABC) and krill herds (KH) (Gandomi and Alavi, 2012; Karaboga and Akay, 2011; Yang, 2010; Yang, 2010). The comparisons indicate that ABC gives better performance than PSO, DE and EA and can effectively be employed for solving engineering problems (Singh, 2009; Karaboga and Basturk, 2008). Similarly, differential evolution (DE) algorithm is based on the idea of improvement of the quality of a member of the population having social differences with other individuals (Storn and Price, 1997). Mine blast algorithm (MBA) and grenade explosion method (GEM) are inspired from the explosions of grenades and mines, respectively (Sadollah et al., 2013; Ahrari and Atai, 2010).

Bee colony optimization algorithm (BCOA) is proposed for the solution of numerical problems like traveling salesman, traffic and transportation problems (Teodorovic and Orco, 2005). Encouraging results in complex engineering problems by using BCOA have been reported (Teodorovic, 2003). Yang suggested virtual bee algorithm (VBA) and proved its performance for the solution of numerical problems with two-dimension (Yang, 2005). BA faces a severe drawback of optimal setting of a number of its parameters before (Pham, 2006).

One of the serious drawbacks of ABC is an insufficiency concerning its search equation, which is sufficiently good during exploration of the search space but comparatively poor at exploitation process (Wei-feng *et al.*, 2013). To address such shortcomings of ABC some useful modifications are available from the past studies. Wei-feng *et al* have proposed an improved variant of ABC based on an orthogonal learning (OL) strategy (Wei-feng *et al.*, 2013). To improve the convergence rate of ABC a global-best artificial bee colony (GABC) is presented (Zhu and Kwong, 2010). Getting inspired by DE, the modified ABC/Best/1 and ABC/Rand/1 are employed to perform local searches (Gao and Liu, 2012). The concepts of the so-far-best information, inertial weight, and acceleration coefficient have been employed in the form of IABC (Improved Artificial Bee Colony) (Li *et al*. 2012). The idea of rosenbrock's rotational method has been hybridized with ABC in (Kang *et al*., 2011). In the onlooker bee phase, a memory board based mechanism for selection of neighbouring solutions has been proposed (Mustafa and Ahmet, 2014).

Unfortunately, the past modifications are not very specific to improve the two very important components of ABC, namely the initialization phase and the scout-bees phase. For improving the initialization process and the scout-bees phase the concepts of reflection and effective radius are incorporated respectively. The resulting method is named as radial artificial bee colony algorithm (RABC). The main objective of the present work is to evolve a better and more effective variant of ABC by improving these two mentioned components.

MARTIALS AND METHODS

Artificial bee colony algorithm: Phases of ABC were conducted as following (Karaboga and Akay, 2011).

Initialization phase: This phase was supposed to begin with a user defined population size which could be varied from problem to problem. Half of the population comprised of employed bees and rest of them were considered as onlooker bees. Each randomly generated location described a food source assigned to an employed bee and was produced by using following equation.

$$x_{i,j} = x_j^{\min} + \lambda\left(x_j^{\max} - x_j^{\min}\right)$$

i=1, 2,…, N and *j=1, 2, …, D*

here $x_{i,j}$ represented the j^{th} dimension or parameter of the i^{th} food source or an employed bee, $x_j^{\max}$ and $x_j^{\min}$ were the bounds on the j^{th} parameters, respectively, λ was a randomly selected number between 0 and 1, N represented the employed bees count and D was the dimensionality of the problem to be optimized. Moreover, in this phase, the resetting of the parameter abandonment counter (AC) for each food source also took place. Thereafter, the following formula was used to calculate the fitness of each food source.

$$\text{fit}_i = \begin{cases} \dfrac{1}{1+f_i} & \text{if } (f_i \geq 0) \\ 1+\text{abs}(f_i) & \text{otherwise} \end{cases} \qquad (1)$$

Where fit_i denoted the fitness of i^{th} employed bee at its relevant food source and f_i was the objective function value of i^{th} food source.

Employed bee phase: In this phase, each food source was improved by waggle dance of corresponding employed bee by using following equation:

$$v_{i,j} = x_{i,j} + \phi\left(x_{i,j} - x_{k,j}\right) \quad i, k \in 1,2,\ldots,N,\ j \in 1,2,\ldots,D \text{ and } i \neq k$$

Here $v_{i,j}$ was the j^{th} component of i^{th} solution vector, $x_{i,j}$ was j^{th} component of i^{th} food source, $x_{k,j}$ was j^{th} component of k^{th} food source and ϕ acted as a randomly selected number between −1 and +1. Moreover, the component (j) and the neighboring candidate solution (k) were selected randomly from the remaining members of the population.

Using equation (1) the fitness of new location $v_{i,j}$ was found and was assigned $x_{i,j}$, provided the fitness of new source was high. The value of AC was reset to zero in case of success and was increased by 1 in case of failure.

Onlooker bee phase: In this phase, the waggle dance of employed bees helped onlookers to get aware of the better positions. Then the onlooker bees were designated to the food sources depending on its probability (p_i) of selection calculated by following equation:

$$p_i = \frac{\text{fit}_i}{\sum_{j=1}^{N} \text{fit}_j}$$

Afterwards, the onlooker bees aimed to seek improvement in the assigned food sources shared by employed bees using waggle dance equation again. If the solution gained by onlooker bee was better than that of the employed bee, the newly found better solution of the onlooker phase was memorized to employed bee and AC was reset.

Scout bee phase: In this phase, the abandon counter with maximum content was matched with pre-defined limit value. If value of AC having maximum content was greater than the limit value, the corresponding employed bee was converted to a scout bee and a new food source was generated randomly. The AC was reset. After generating new solution for itself, the scout bee returned to instatement as employed bee.

Proposed Modifications: In original format of ABC, a random search was made in the search space for selection of food sources. But, this technique of initialization did not grantee the proper exploration of the search space. To respond this challenge the algorithmic alteration as per modification # 1 for initialization of N food sources in the search space was presented.

Modification # 1:

step-1 i=1

step-2 for j= 1,2,3…,N/2

x_i = L + rand(0,1)(U-L)

x_{i+1} = x_i + 2((L+U)/2-xi);

step-3 i = i+ 2, go to step-4

step-4 if i<=N go to step-2 else stop

The third line of step-2 performed the function of reflection about the center of the search space which did not permit a solution and its reflected point to take place in the same sub region of search space.

Modification 2: Second modification dealt with the determination of appropriate position for a scout bee whose budget of waggle dance was exhausted. For the said employed bee, a scout bee explored the search space while capturing a new position calculated equation 2.

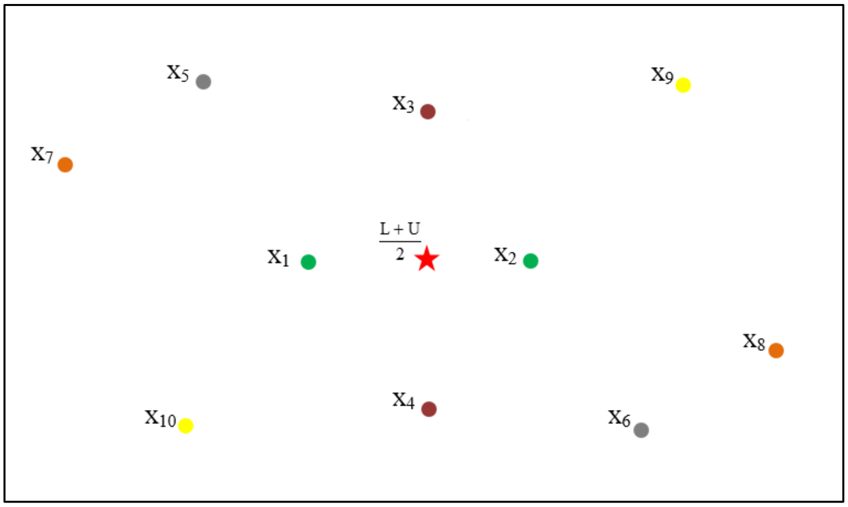

Fig.1: N initial locations are distributed randomly in 2-dimensional search space

$$x_{new} = x^b + Rad \times rand(-1,1) \qquad (2)$$

Here $Rad = \max\left\{ \left|x_i^b - l_i\right|, \left|x_i^b - u_i\right| : 1 \le i \le n \right\}$ (3)

Further rand (-1,1) denotes a vector of random numbers produced in the interval (-1, 1).

Due to impact of eqn.(6), each scout bee was launched in a radius calculated by equation 3 around the current best solution in a random direction. RABC algorithm was evolved by embedding modifications 1 and 2 into the original ABC. The complete RABC algorithm was stated as under.

RABC Algorithm

Step 1: Parameters were selected.
Step 2: Initialize population was generated by using modification # 1
Step 3: Employed Bees Phase was activated.
Step 4: Onlooker Bees phase was executed.
Step 5: Parameter AC was checked and scout bees were launched using modification #2.

RESULTS AND DISCUSSION

Un-constrained benchmark test functions: The unconstrained optimization problems considered for comparison involved Griewank's function (f_1), Rastrigin's function (f_2), Rosenbrock function (f_3), Ackley function (f_4) and Schwefel's function (f_5) with search domains [−600, 600], [−15, 15], [−15, 15], [−32.768, 32.768] and [−500, 500] respectively. All the benchmarks had a global minimum value of 0.

$$f_1(\vec{x}) = \frac{1}{4000}\left(\sum_{i=1}^{D} x_i^2\right) - \left(\prod_{i=1}^{D}\cos\left(\frac{x_i}{\sqrt{i}}\right)\right) + 1$$

$$f_2(\vec{x}) = \sum_{i=1}^{D}\left(x_i^2 - 10\cos(2\pi x_i) + 10\right)$$

$$f_3(\vec{x}) = \sum_{i=1}^{D} 100\left(x_i^2 - x_{i+1}^2\right)^2 + \left(1 - x_i\right)^2$$

$$f_4(\vec{x}) = 20 + e - 20e^{\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right)} - e^{\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)}$$

$$f_5(\vec{x}) = D * 418.9829 + \sum_{i=1}^{D} - x_i \sin\left(\sqrt{\left|x_i\right|}\right)$$

The problems f_1 – f_5 were solved by vortex search (VS) algorithm, GA, PSO, PS-EA and ABC algorithms (B. Dogˇan and T. Ölmez, 2015; Karaboga and Akay, 2011; Srinivasan and Seow, 2003).

The results, presented in Table 8, were directly extracted from the respective references of the algorithms. For 10 and 20-dimensional f_1 function, RABC produced the best results in comparison with those of GA, PSO, PS-EA and PSO whereas on its 30-dimensional case only ABC was little better. The mean best values found by VS and an orthogonal learning based global best artificial bee colony (OGABC) were 0.032798017 and 9.85E-04 respectively (B. Dogˇan and T. Ölmez, 2015; Gao et al, 2013). The results of RABC were better than these results. From table 8 it could be noticed that RABC outperformed the algorithms GA, PSO and PS-EA on all the test problems for dimension 10, 20 and 30.

For 30-dimensional f_2 function, the mean best values found by RABC was 4.14e-13 which was better than the mean best values 8.50E-13, 1.26E-11 found by ABC and OGABC respectively (Gao et al, 2013; Karaboga and Akay, 2011). Similarly, for rosenbrock function (f_3), RABC stood 1st for dimension 10 and 30 and also found comparable mean solution to that of ABC. The final mean best solution found by RABC for f_3 was also better than those of references (B. Dogˇan and T. Ölmez, 2015; Gao et al, 2013; Srinivasan and Seow, 2003).

Table 1: Results obtained by GA, PSO, PS-EA, ABC and RABC Algorithms.

Algorithm		GA		PSO		PS-EA		ABC		RABC	
Function	**Dim**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**
Griewank	10	0.050228	0.029523	0.079393	0.033451	0.22236	0.0781	0.001634	0.003939	**0.000251**	**0.001373**
	20	1.0139	0.026966	0.030565	0.025419	0.59036	0.2030	0.000412	0.002255	**1.38e-10**	**6.98e-10**
	30	1.2342	0.11045	0.011151	0.014209	0.8211	0.1394	**0.000493**	0.002696	0.000829	**0.002564**
Rastrigin	10	1.3928	0.76319	2.6559	1.3896	0.43404	0.2551	**0**	**0**	**0**	**0**
	20	6.0309	1.4537	12.059	3.3216	1.8135	0.2551	**1.8e-014**	**2.29e-14**	2.65e-14	2.68e-14
	30	10.4388	2.6386	32.476	6.9521	3.0527	0.9985	8.50e-13	1.84e-12	**4.14e-13**	**4.31e-13**
Rosenbrock	10	46.3184	33.8217	4.3713	2.3811	25.303	29.7964	0.06202	0.070766	**0.024981**	**0.022851**
	20	103.93	29.505	77.382	94.901	72.452	27.3441	**0.038573**	**0.040478**	0.080213	0.14019
	30	166.283	59.5102	402.54	633.65	98.407	35.5791	0.15941	0.26833	**0.10659**	**0.15093**
Ackley	10	0.59267	0.22482	9.84e-13	9.62e-13	0.19209	0.1951	**7.99e-15**	1.86e-15	8.23e-15	**1.59e-15**
	20	0.92413	0.22599	1.177e-6	1.584e-6	0.32321	0.097353	3.22e-14	4.38e-15	**2.99e-14**	**4.57e-15**
	30	1.0989	0.24956	1.491e-6	1.861e-6	0.3771	0.098762	**4.17e-13**	**1.77e-13**	4.38e-13	2.09e-13
Schwefel	10	1.9519	1.3044	161.87	144.16	0.32037	1.6185	**-4189.82**	**2.35e-12**	**-4189.82**	2.56e-12
	20	7.285	2.9971	543.07	360.22	1.4984	0.84612	**-8379.65**	6.307e-5	**-8379.65**	**2.47e-10**
	30	13.5346	4.9534	990.77	581.14	3.272	1.6185	-12557.5	36.1145	-12541.6	58.6867

On Ackley function, the mean solution attained by RABC for dimension 20 was the best of all its competitors whereas the mean solutions for dimension 10 and 30 were very close to those of ABC but surely better than all the remaining algorithms. On Schwefel function, the mean best solution of RABC was better than those of its competitors for dimension 10 and 20 but was little worse for dimension 30. As an overall remark, we concluded that in most of the cases RABC was superior to its competitors.

Engineering design constrained problems

Pressure vessel optimum design problem: The pressure vessel optimum design problem involved the minimization of the entire cost that comprised of the material cost, welding and forming costs (Kannan and Kramer, 1994). The geometry of the problem was presented in figure 2. The mathematical model was of the form:

Minimize

$$f(\underline{x}) = 0.6224\ x_1 x_4 x_3 + 3.1661\ x_4 x_1^2 + 19.84\, x_3 x_1^2 + 1.7781\ x_2 x_3^2$$

Subject to $g_1(\underline{x}) = 0.0193 x_3 - x_1 \le 0$

$$g_2(\underline{x}) = 0.00954 x_3 - x_2 \le 0$$

$$g_3(\underline{x}) = 1296000 - \frac{4}{3}\pi x_3^3 - \pi x_3^2 x_4 \le 0$$

$$g_4(\underline{x}) = -240 + x_4 \le 0$$

$$x_1, x_2 \in [0,100]\ ;\ x_3, x_4 \in [10, 200]$$

Table 2 witnessed that RABC outperformed the results of a number of past approaches (Coello, 2000; Coello and Montes, 2002; Parsopoulos and Vrahatis, 2005; Mezura and Coello, 2005; Akay and Karaboga, 2010). Their optimization results (statistical results) were taken from their respective references. RABC found the smallest optimum value which was much better than each of Ndirituand Daniell, Wu and Chow, Sandgren, Fu and Khalil.

Table 2: Comparison of results for pressure vessel problem.

D.V	Ndirituand Daniell	Wu and Chow	Sandgren	Fu	Khalil	ABC	RABC
x_1	1.125	1.125	1.125	1.125	1.125	1.1418	1.125
x_2	0.625	0.625	0.625	0.625	0.625	0.625	0.625
x_3	58.2209	58.1978	48.97	48.3807	58.2367	58.6513	58.2902
x_4	44.086	44.2930	106.72	111.7449	44.0247	41.7345	43.6925
$g_1(x)$	-0.00133663	-0.00178246	-0.179879	-0.19125249	-0.00103169	-0.00982991	-8.6e-007
$g_2(x)$	-0.069572614	-0.069792988	-0.1578262	-0.163448122	-0.069421882	-0.065466598	-0.068911492
$g_3(x)$	-127.6084334	-974.1573	97.9031762	-72.9716183	-402.51442	-150.89312	-0.94789
f(x)	7202.517	7207.497	7982.5	8048.619	7204.32	7197.9114	**7197.7299**

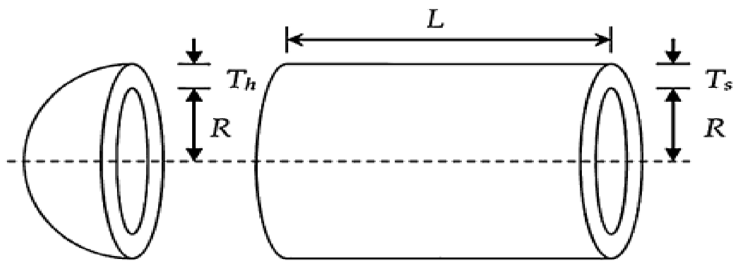

Fig. 2: Geometry of pressure vessel.

Figure 3 presented the iterative convergence curve of the so far best function values for the pressure vessel problem.

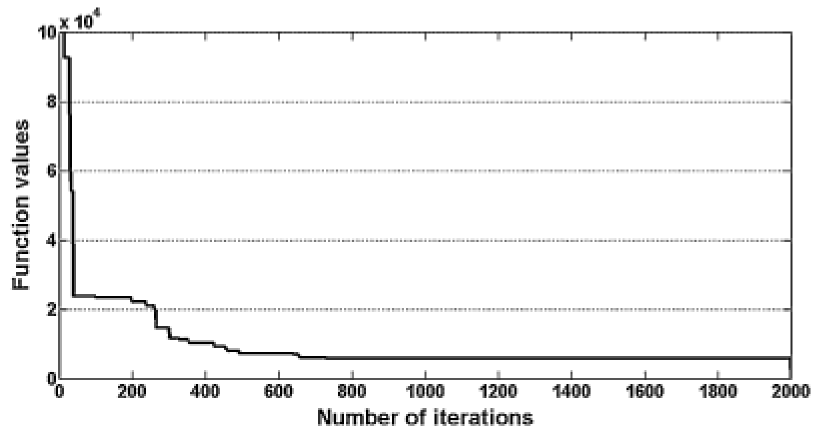

Fig. 3. RABC convergence progress for the pressure vessel optimum design problem.

Welded beam optimum design problem: Welded beam problem was developed by Coello (Coello, 2000). This problem involved four design variables h(x_1), *l*(x_2), t(x_3), and b(x_4) as given in Fig. 4. The mathematical model was presented as following:

Minimize

$$f(\underline{x})=0.04811x_4x_3(x_2+14)+1.10471x_2x_1^2$$

Subject to : $g_1(\underline{x})=-\tau_{\max}+\tau(\underline{x})\le 0$

$$g_2(\underline{x})=-\sigma_{\max}+\sigma(\underline{x})\le 0$$

$$g_3(\underline{x})=-x_4+x_1\le 0$$

$$g_4(\underline{x})=-5+0.04811x_3x_4(x_2+14)+0.10471x_1^2\le 0$$

$$g_5(\underline{x})=-x_1+0.125\le 0$$

$$g_6(\underline{x})=-\delta_{\max}+\delta(\underline{x})\le 0$$

$$g_7(\underline{x})=-P_c(\underline{x})+P\le 0$$

$x_1,x_4\in[0.1,2]$ and $x_2,x_3\in[0.1,10]$

Where

$$\tau(x)=\sqrt{(\tau')^2+2\tau'\tau''\frac{x_2}{2R}+(\tau'')^2},$$

$$\tau'=\frac{p}{\sqrt{2}x_1x_2},\quad \tau''=\frac{MR}{J},$$

$$M=P\left(L+\frac{x_2}{2}\right),$$

$$J = 2\sqrt{2}x_1x_2\left[\frac{x_2^2}{12}+\left(\frac{x_3+x_1}{2}\right)^2\right],$$

$$R = \frac{1}{2}\sqrt{(x_1+x_3)^2+x_2^2},$$

$$\sigma(x)=\frac{6PL}{x_4x_3^2}, \qquad \delta(x)=\frac{4PL^3}{Ex_3^3x_4},$$

$$P_c(x)=\frac{4.013E\sqrt{x_3^2x_4^6/36}}{L^2}\times\left(1-\frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$$

P=6000 lb, L=14 in, E=30000000 psi, G=12000000psi, τ_{max}= 13600 psi, σ_{max}=30,000 psi, δ_{max}= 0.25 in

The optimization methods which have been earlier applied to this problem include GA4, CAEP, CPSO, HPSO, HGA, (Mezura and Coello, 2006) and ABC. The best solutions achieved by different algorithms were presented in Table 3.

Table 3: Comparison of result for welded beam problem.

D.V	HGA	GA4	CAEP	HPSO	CPSO	ABC	RABC
x_1	0.2057	0.205986	0.205700	0.20573	0.202369	0.2955	0.20583
x_2	3.4705	3.471328	3.470500	3.470489	3.544214	2.4168	3.4679
x_3	9.0366	9.020224	9.036600	9.036624	9.048210	8.2386	9.0377
x_4	0.2057	0.206480	0.205700	0.20573	0.205723	0.29592	0.20583
g_1	1.988676	−0.103049	1.988676	−0.025399	−13.655547	-4.693309	-4.6933098
g_2	4.481548	−0.231747	4.481548	−0.053122	−78.814077	-4907.18509	-4907.18509
g_3	0	−0.0005	0	0	−0.00335	-0.00042	-0.000420
g_4	−3.433213	−3.430044	−3.433213	−3.432981	−3.424572	-0.2367340	-10771.560
g_5	−0.080700	−0.080986	−0.080700	−0.08073	−0.077369	-10.771560	-3.0653212
g_6	−0.235538	−0.235514	−0.235538	−0.235540	−0.235595	-3.065322	-0.1705
g_7	2.603347	−58.646888	2.603347	−0.031555	−4.472858	-0.1705	-0.236735
f(x)	1.725852	1.728226	1.724952	**1.7249**	1.728024	1.7673	**1.7249**

Table 3 validated that the objective function value found by RABC was better than that of each of GA4, HGA, CPSO, CAEP, and original ABC and little higher the HPSO.

All the constraints were satisfied in RABC, ABC, HPSO, CPSO and GA4. However the solutions found by HGA and CAEP were infeasible. The results of all the competing algorithms were extracted from the reference concerning to Mine Blast Algorithm (MBA) (Sadollah et al, 2013).

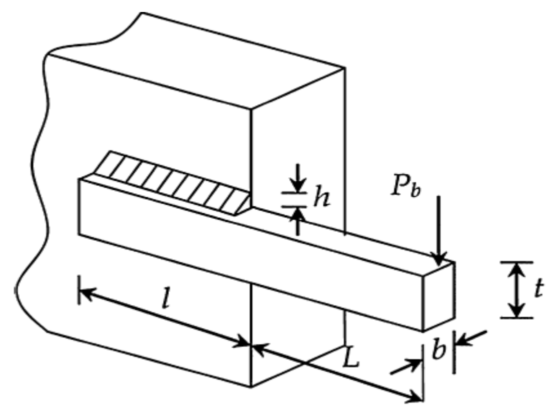

Fig. 4. Welded Beam Design Problem.

Table4: Comparison of results for the welded beam problem taken from literature.

Method	Worst	Mean	Best	SD
GA4	1.993408	1.792654	1.728226	0.0747
CAEP	3.179709	1.971809	1.724852	0.443
CPSO	1.782143	1.748831	1.728024	0.0129
HPSO	1.814295	1.749040	1.724852	0.0401
HGA	1.824105	1.768158	1.733461	0.0221
ABC	2.0706	1.8781	1.7673	0.073555
RABC	1.7277	**1.7254**	1.7249	**0.00063131**

From table 4 one could conclude that the best solution of f(x) = 1.7249 attained by RABC was better than all the remaining algorithms.

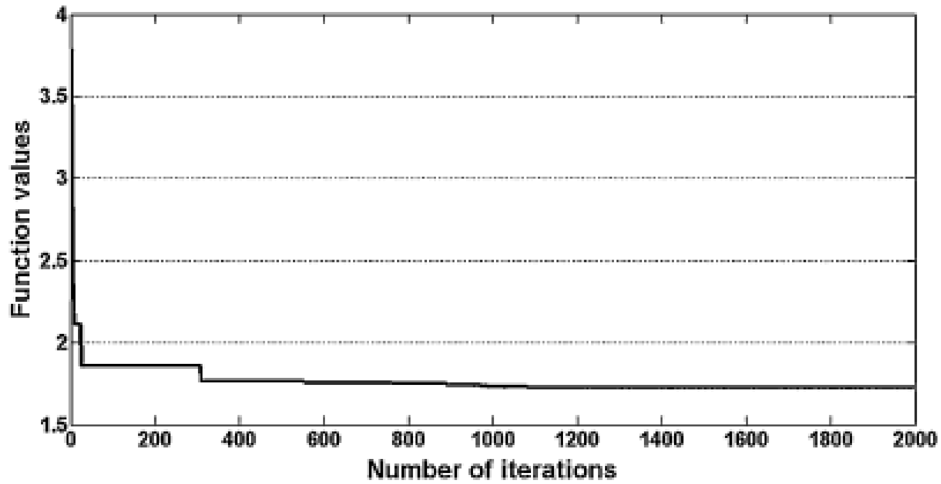

Fig. 5. Convergence curve of RABC for the welded beam problem.

Fig. 5 showed the convergence progress of RABC on welded beam optimum design problem.

Speed reducer optimum design problem: Speed reducer design problem was designed for minimization of the weight of speed reducer under the constraints on bending stresses of the surface, gear teeth and shafts along with their transverse deflections (Akay and Karaboga, 2010). Figure 6 showed the geometry and all the design variables of the problem. There were 11 constraints due to which problem became highly complex (Kuang *et al.*, 1998). Mathematical form of the problem was as given below.

Minimize

$$f(\underline{x}) = 0.7854x_1x_2^2\left(\frac{10}{3}x_3^2 - 43.0934 + 14.9334x_3\right) + 7.4777(x_6^3 + x_7^3) - 1.508x_1(x_6^2 + x_7^2) +$$

$$0.7854(x_4x_6^2 + x_5x_7^2)$$

Subject to:

$$g_1(\underline{x}) = 27 - x_1x_2^2x_3 \le 0, \quad g_2(x) = 397.5 - x_1x_2^2x_3^2 \le 0$$

$$g_3(x) = \frac{1.93x_4^3}{x_2x_6^4x_3} - 1 \le 0, \quad g_4(x) = \frac{1.93x_5^3}{x_2x_7^4x_3} - 1 \le 0$$

$$g_5(x) = \frac{\left[745(x_4/x_2x_3)^2 + 16.9\times10^6\right]^{1/2}}{110x_6^3} - 1 \le 0$$

$$g_6(x) = \frac{\left[745(x_5/x_2x_3)^2 + 157.5\times10^6\right]^{1/2}}{85x_7^3} - 1 \le 0$$

$$g_7(x) = \frac{x_2x_3}{40} - 1 \le 0, \quad g_8(x) = \frac{5x_2}{x_1} - 1 \le 0$$

$$g_9(x) = \frac{x_1}{12x_2} - 1 \le 0, \quad g_{10}(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \le 0$$

$$g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \le 0$$

Where

$2.6 \le x_1 \le 3.6$, $0.7 \le x_2 \le 0.8$, $17 \le x_3 \le 28$, $7.3 \le x_4 \le 8.3$, $7.3 \le x_5 \le 8.3$, $2.9 \le x_6 \le 3.9$, $5.0 \le x_7 \le 5.5$

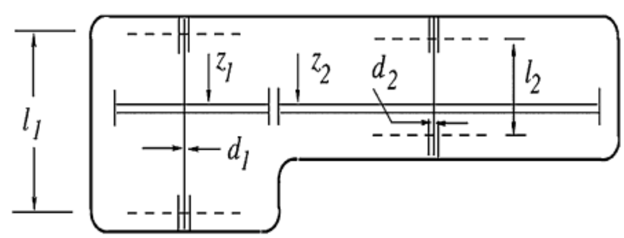

Fig. 6. Speed reducer design problem.

The comparison with earlier methods regarding best solution for this problem was given in Table 5. The computed results of RABC were compared with those of the methods including DELC, PSO-DE, DEDS, ABC, HEAA and a modified differential evolution (MDE). Tables 5 and 6 showed the comparisons.

This table5 reveals that the value of objective function of RABC is better than the DEDS, DELC, HEAA, MDE, ABC and little higher the PSO-DE.

From table 6 it was evident that the best function value of f(x) = 2994.4701 with very small standard deviation of **1.1198E-11** found by RABC was better than its competitors. The small deviation expressed that RABC was highly consistent. Fig. 7 displayed the function values versus the number of iterations for the welded beam design problem.

Table 5: Comparison of the best results for speed reducer optimum design problem obtained from various previous studies.

D.V	DEDS	PSO-DE	DELC	ABC	HEAA	MDE	RABC
x_1	3.5E+09	3.5000000	3.5E+09	3.5	3.500022	3.500010	**3.50008**
x_2	0.7E+09	0.700000	0.7 E+09	0.7	0.70000039	0.70000	**0.7**
x_3	17	17.000000	17	17	17.000012	17	**17**
x_4	7.3E+09	7.300000	7.3 E+09	7.3	7.300427	7.300156	**7.3**
x_5	7.715319	7.800000	7.715319	7.71532	7.715377	7.800027	**7.71533**
x_6	3.350214	3.350214	3.350214	3.35021	3.350230	3.350221	**3.35021**
x_7	5.286654	5.2866832	5.286654	5.28665	5.286663	5.286685	**5.28665**
f(x)	2994.471066	2996.348167	2994.481066	2994.4711	2994.499107	2996.556689	**2994.4701**

Table 6: Comparison of statistical results given by different methods for speed reducer design problem.

Method	Best	Mean	Worst	SD
PSO-DE	**2996.348167**	2996.348174	2996.348204	**6.4E−12**
DELC	2994.481066	2994.471066	2994.471066	1.9E−11
DEDS	2994.471066	2994.471066	2994.471066	3.6E−11
HEAA	2994.499107	2994.613368	2994.752311	7.0E−02
MDE	2996.556689	2996.367220	N.A	8.2E−03
ABC	2994.4711	2994.4738	2994.5009	0.00843
RABC	**2994.4701**	2994.4711	2994.4711	**1.1198E-11**

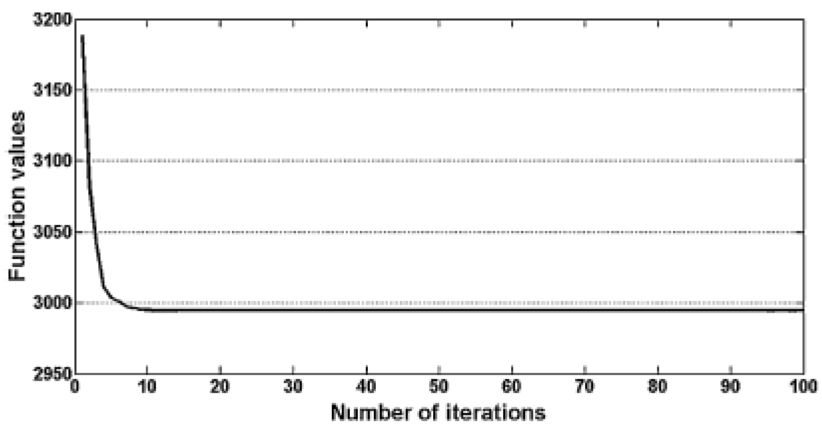

Fig. 7. Function values versus number of iterations for the speed reducer problem.

Conclusion: Numerical comparisons of the study have witnessed that the RABC found better solution to the unconstrained as well as constrained optimization problems in comparison with other meta-heuristic optimizers. However, its efficiency and quality of solutions was dependent on the complexity and nature of the test problem.

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