FOURTH ORDER COMPACT METHOD FOR ONE DIMENSIONAL HOMOGENEOUS TELEGRAPH EQUATION

Z. U. A. Zafar, M. O. Ahmad and A. Pervaiz*

Faculty of Engineering, University of Central Punjab, Lahore, Pakistan
Department of Mathematics, University of Engineering & Technology Lahore, Pakistan
Corresponding Author: zainulabadin1@hotmail.com

ABSTRACT: Many boundary value problems that arise in real life situations defy analytical solutions; hence numerical techniques are the best source for finding the solution of such equations. In this paper, a compact method for homogeneous telegraph equation is developed, and comparison of the compact method with the second order scheme is also given. We have obtained results both numerically and graphically. We used FORTRAN 90 for the calculations of the numerical results and MS office for the graphical comparison.

Key words: Finite Difference Method, Fourth Order Compact Method, Telegraph Equation

INTRODUCTION

The compact method is a new difference approximation. It is the fourth order approximation using only three grid points; whereas standard fourth order centered difference approximation requires five points, so in this method we have a higher order accuracy using fewer grid points. In (ORSZAG; 1974) a compact formula was mentioned and this method was used in that manner by Ciment and Leventhal (1978) for hyperbolic problems. Telegraph equation occurs in the study of transmission of electrical signals in a cable line and wave phenomenon. Biologists come across these equations in the study of pulses blood flow in arteries and in one dimensional haphazard movement of bugs along a hedge. Also the transmission of acoustic waves in Darcy type permeable medium and analogus flows of viscous Maxwell fluids are presently a few of the phenomenon by Telegraph equation.

Let us consider the second order one-dimensional linear hyperbolic equation

$$\alpha \frac{\partial^2 u(x,t)}{\partial t^2} + \beta \frac{\partial u(x,t)}{\partial t} + \gamma u(x,t) = c^2 \quad (1)$$

with the following initial conditions

(2)

(3)

and with the boundary conditions

(4)

(5)

$for \qquad 0 \le x \le l, \quad t > 0$

Eq. (1) is referred to as the second order linear homogeneous Telegraph Equation with constant

coefficients. In eq. (1), is distance and is time. For

, , and , equation (1)

represents a damped wave equation and for

are non negative integers then it is called telegraph equation.

Finite difference scheme: To develop the finite difference scheme for eq. (1), select an integer m **AND THE VALUES OF** t **FROM** 0 **TO** ∞ **THEN THE MESH POINTS** (x_i, t_n) **ARE**

x_i for

t_n for

At any interior grid points , then the Hyperbolic Homogeneous Telegraph eq. (1) becomes

$$\alpha \frac{\partial^2 u(x_i,t_n)}{\partial t^2} + \beta \frac{\partial u(x_i,t_n)}{\partial t} + \gamma u(x_i,t_n) = c^2 \frac{\partial^2 u(x_i}{\partial x^2} \quad (6)$$

The method is obtained by using the central difference approximation for the first and second order partial derivatives.

So that eq. (6) becomes

$$\frac{\alpha}{(\Delta t)^2}\left(u_i^{n+1}-2u_i^n+u_i^{n-1}\right)-\frac{\alpha(\Delta t)^2}{12}\frac{\partial^4 u(x_i,\mu_n)}{\partial t^4}+$$

$$\frac{\beta}{2(\Delta t)}\left(u_i^{n+1}-u_i^{n-1}\right)-\frac{\beta(\Delta t)^2}{6}\frac{\partial^3 u(x_i,\mu_n)}{\partial t^3}+\gamma u_i^n=\frac{c}{(\Delta}$$

where

Neglecting the truncation error leads to the difference equation.

$$\frac{\alpha}{(\Delta t)^2}\left(u_i^{n+1}-2u_i^n+u_i^{n-1}\right)+\frac{\beta}{2(\Delta t)}\left(u_i^{n+1}-u_i^{n-1}\right)+\gamma u$$

$$\frac{c^2}{(\Delta x)^2}\left(u_{i+1}^n+u_{i-1}^n\right)=\left(\frac{\alpha}{(\Delta t)^2}+\frac{\beta}{2(\Delta t)}\right)u_i^{n+1}+$$

$$\left(\gamma-\frac{2\alpha}{(\Delta t)^2}+\frac{2c^2}{(\Delta x)^2}\right)u_i^n+\left(\frac{\alpha}{(\Delta t)^2}-\frac{\beta}{2(\Delta t)}\right)u_i^{n-1}$$

Taking

$\Big(\frac{}{(\Delta}$. $\left(\gamma-\frac{2\alpha}{(\Delta t)^2}\right.$. and $\Big(\frac{}{(}$

So

$$\frac{c^2}{(\Delta x)^2}\left(u_{i+1}^n+u_{i-1}^n\right)=\lambda_1 u_i^{n+1}+\lambda_2 u_i^n+\lambda_3 u_i^{n-1}$$

$$\lambda_1 u_i^{n+1}=\frac{c^2}{(\Delta x)^2}\left(u_{i+1}^n+u_{i-1}^n\right)-\lambda_2 u_i^n-\lambda_3 u_i^{n-1}$$

$$u_i^{n+1}=\frac{c^2}{\lambda_1(\Delta x)^2}\left(u_{i+1}^n+u_{i-1}^n\right)-\frac{\lambda_2}{\lambda_1}u_i^n-\frac{\lambda_3}{\lambda_1}u_i^{n-1}$$

By letting , and

So

$$u_i^{n+1}=\Lambda\left(u_{i+1}^n+u_{i-1}^n\right)+\Psi u_i^n+\Phi u_i^{n-1}$$

$$u_i^{n+1}=\Psi u_i^n+\Lambda u_{i+1}^n+\Lambda u_{i-1}^n+ \qquad (7)$$

This equation holds for each i: The boundary conditions give

(8)

for each .

And the initial condition implies that

(9)

for i:

Writing in matrix form for i: we have

$$\begin{bmatrix}u_1^{n+1}\\u_2^{n+1}\\ \vdots\\ \vdots\\ u_{m-1}^{n+1}\end{bmatrix}=\begin{bmatrix}\Psi&\Lambda&0&&0\\ \Lambda&\Psi&\Lambda&&0\\ 0&\Lambda&\ddots&\ddots&0\\ &&\ddots&\Psi&\Lambda\\ 0&&0&\Lambda&\Psi\end{bmatrix}\begin{bmatrix}u_1^n\\u_2^n\\ \vdots\\ \vdots\\ u_{m-1}^n\end{bmatrix}+\Phi\begin{bmatrix}u_1^{n-}\\u_2^{n-}\\ \vdots\\ \vdots\\ u_{m-}^{n-}\end{bmatrix} \qquad (10)$$

Equations (7) and (8) imply that the time steps requires values from the and time steps. This produces a minor starting problem since values of

which is needed, in equation (7) to compute must be obtained from the initial value condition.

$$u_t|_i^0=g(x_i), \qquad 0\le x\le l$$

A better approximation can be obtained rather easily, particularly when the second derivative of at can be determined.

Consider the Taylor Series

$$u_i^{n+1}=u_i^n+k\,u_t|_i^n+\frac{k^2}{2}u_{tt}|_i^n+\frac{k^3}{6}u_{ttt}|_i^n+\frac{k^4}{24}u_{tttt}|_i^n+\frac{k^5}{120}u_{tttt}$$

$$\frac{u_i^{n+1}-u_i^n}{k}=u_t|_i^n+\frac{k}{2}u_{tt}|_i^n+\frac{k^2}{6}u_i^{(3)n}(x_i,\mu_n)$$

For we have

$$\frac{u_i^1-u_i^0}{k} = u_t|_i^0 + \frac{k}{2}\, u_{tt}|_i^0 + \frac{k^2}{6}\, u_i^{(3)0} \quad (11)$$

for some in and suppose the homogeneous telegraph equation also holds on the initial line. That is by equation (2)

$$u_{tt}|_i^0 = \frac{c^2}{\alpha}\, f''(x_i) - \frac{\beta}{\alpha} u_t|_i^0 - \frac{\gamma}{\alpha} u_i^0$$

Substituting this value in eq.(11), we get

$$\frac{u_i^1 - u_i^0}{k} = u_t|_i^0 + \frac{k}{2}\left(\frac{c^2}{\alpha}\, f''(x_i) - \frac{\beta}{\alpha} u_t|_i^0 - \frac{\gamma}{\alpha} u_i^0\right) + \frac{k^2}{6}\, u_i^{(3)0}(x_i$$

but

$$u_t|_i^0 = g(x_i)$$

So on simplifying we get

$$u_i^1 = \frac{k^2c^2}{2\alpha} f''(x_i) + \left(k - \frac{\beta k^2}{2\alpha}\right) g(x_i) + \left(1 - \frac{\gamma k^2}{2\alpha}\right) u_i^0$$

This is an approximation with local truncation error

for each

Now from the difference equation

$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$

$$u_i^1 = \frac{k^2c^2}{2\alpha}\left(\frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}\right) + \left(k - \frac{\beta k^2}{2\alpha}\right) g(x_i) +$$

$$u_i^1 = \frac{k^2c^2}{2\alpha h^2}(f(x_{i+1}) + f(x_{i-1})) + \left(k - \frac{\beta k^2}{2\alpha}\right) g(x_i) + \left(1 - \frac{\gamma k^2}{2\alpha}\right.$$

But and letting , we have

$$u_i^1 = \frac{\lambda^2}{2\alpha}(f(x_{i+1}) + f(x_{i-1})) + \left(k - \frac{\beta k^2}{2\alpha}\right) g(x_i) + \left(1 - \frac{\gamma k^2}{2\alpha} - \frac{\lambda^2}{\alpha}\right) f(x_i) \quad (12)$$

for each i Note here and

.

Compact scheme for telegraph equation: To develop this scheme for the second order linear hyperbolic homogeneous Telegraph eq. (1) with and are given functions. This Compact method approximates eq. (1) by two difference equations of fourth order using only three grid points say and . Let us denote first and second derivatives of with respect to by two fictitious alphabets respectively.

$$(13)$$

$$u_{xx}(x,t) = S$$

We shall first develop a relationship between the values of and . Since , it is clear that

$$u_{i+1}^n = u_{i-1}^n + \int_{i-1}^{i+1} F(\xi, l)\, d\xi$$

Approximating this integral by Simpson's Rule and rearranging it, so we have

$$u_{i+1}^n = u_{i-1}^n + \frac{h}{3}(F_{i-1}^n + 4F_i^n + F_{i+1}^n) + \frac{h^5}{90}\, \frac{\partial^4 F(\xi, l)}{\partial x^4}$$

Thus to fourth order, we have

$$(F_{i-1}^n + 4F_i^n + F_{i+1}^n) + \frac{3}{h}(u_{i-1}^n - u_{i+}^n \quad (14)$$

So we have a relationship between and . This is the first difference equation.

In order to obtain the second equation, we start by evaluating (1) at the mid point . Then eq. (1) becomes

$$\alpha u_{tt}|_i^n + \beta u_t|_i^n + \gamma \quad (15)$$

We now require an expression for . If we express and in Taylor series about the point and adding the results we get

$$u_{i+1}^n + u_{i-1}^n = 2u_i^n + h^2\, S|_i^n + \frac{h^4}{12} u_{xxxx}|_i^n + \frac{h^6}{360} u_{xxxxxx}(\xi, l)|_i^n$$

(16)

where we have replaced with . If we carry out the

same procedure for then we have

$$F_{i+1}^n - F_{i-1}^n = 2\,h\,S|_i^n + \frac{h^3}{3}u_{xxxx}|_i^n + \frac{h^5}{60}u_{xxxxxx}(\xi, l)|_i^n$$

(17)

We now eliminate from these two equations and after rearranging, we get the following expression for $S|_i^n$, (See Ozair, A compact method for heat equation, with equations (8 , where tells the equation number in Ozair's paper).

We now substitute the expression for into (15) and rearrange to get the following second difference equation of fourth order.

$$\alpha u_{tt}|_i^n + \beta u_t|_i^n = \frac{2\,c^2}{h^2}(u_{i+1}^n + u_{i-1}^n) - \left(\gamma + \frac{4\,c^2}{h^2}\right)u_i^n - \frac{c^2}{2h}(F_{i+1}^n - F_{i-1}^n)$$

(18)

We have now replaced (1) by two difference equations (14) and (18). Now we have to look at the boundaries. Let us first consider the left boundary condition i.e., at and denotes the points by . The first difference equation we get from the boundary condition is

(19)

In order to get the second equation, we start with the differential equation at the point and .

$$c^2 S|_0^n = \alpha\, u_{tt}|_0^n + \beta \quad (20)$$

$$c^2 S|_1^n = \alpha\, u_{tt}|_1^n + \beta \quad (21)$$

From (See Ozair, A compact method for heat equation, with equation and), we have the following expressions for and .

$$S|_0^n = \frac{1}{2h^2}(-23u_0^n + 16u_1^n + 7u_2^n) - \frac{1}{h}(6F_0^n + 8F_1^n + F_2^n)$$

(22)

$$S|_1^n = \frac{2}{h^2}(u_0^n - 2u_1^n + u_2^n) - \frac{1}{2h}(F_2^n \quad (23)$$

Finally we have from (14)

$$(F_0^n + 4F_1^n + F_2^n) + \frac{3}{h}(u_0^n \text{ -} \quad (24)$$

So we have five equations (20) to (24). If we eliminate , , and from these five equations, we get the second difference equation, valid at .

$$\left(\frac{12c^2}{h^2}+\gamma\right)u_0^n-\left(\frac{12c^2}{h^2}+\gamma\right)u_1^n+\frac{6c^2}{h}F_0^n+\frac{6c^2}{h}F_1^n=\alpha(u_{tt}|_1^n-u \tag{25}$$

In a similar manner, we can derive the following difference equation for and at , i.e. at the right boundary point.

(26)

$$\left(\frac{12c^2}{h^2}+\gamma\right)u_{m-1}^n-\left(\frac{12c^2}{h^2}+\gamma\right)u_m^n+\frac{6c^2}{h}F_{m-1}^n+\frac{6c^2}{h}F_m^n=\alpha(u_{tt}|_m^n-u_{tt}|_{m-1}^n)+\beta(u_t|_m^n \tag{27}$$

Thus for each point, we have two difference equations. If we write them all together, we have the following Fourth Order

Compact Scheme for .

$$u_0^n=0$$

$$\left(\frac{12c^2}{h^2}+\gamma\right)u_0^n-\left(\frac{12c^2}{h^2}+\gamma\right)u_1^n+\frac{6c^2}{h}F_0^n+\frac{6c^2}{h}F_1^n=\alpha(u_{tt}|_1^n-u_{tt}|_0^n)+\beta(u_t|_1^n-u_t|_0^n)$$

$$\frac{2c^2}{h^2}(u_{i+1}^n+u_{i-1}^n)-\left(\gamma+\frac{4c^2}{h^2}\right)u_i^n-\frac{c^2}{2h}(F_{i+1}^n-F_{i-1}^n)=\alpha u_{tt}|_i^n+\beta u_t|_i^n$$

$$u_m^n=0$$

$$\left(\frac{12c^2}{h^2}+\gamma\right)u_{m-1}^n-\left(\frac{12c^2}{h^2}+\gamma\right)u_m^n+\frac{6c^2}{h}F_{m-1}^n+\frac{6c^2}{h}F_m^n=\alpha(u_{tt}|_m^n-u_{tt}|_{m-1}^n)+\beta(u_t|_m^n-u_t|_{m-1}^n)$$

The superscript is used to denote the time grid lines.

Accuracy of the scheme: Next, we compare the accuracy of the method with the standard five point centered difference scheme of the fourth order. The

relation between and in this method is

$$\frac{1}{6}F_{i-1}+\frac{2}{3}F_i+\frac{1}{6}F_{i+1}=\frac{1}{2h}(u_{i+1}-u_{i-1})$$

and the relation between and in this method is

$$\frac{1}{12}S_{i-1}+\frac{5}{6}S_i+\frac{1}{12}S_{i-1}=\frac{1}{h^2}(u_{i+1}-2u_i+u_{i-1})$$

The accuracy of this scheme is easily obtained by Taylor expansions of the above equations. The consequential truncation error is

$$F_i = u$$

and

$$S_i = u_i'' - \left(\frac{1}{240}\right)h^4\, u^{(6)}$$

The usual five-point fourth order approximations for

and are

$$F_i = \frac{1}{12h}(-u_{i+2}+8u_{i+1}-8u_{i-1}+u_{i-2})$$

and

$$S_i = \frac{1}{12h^2}(-u_{i+2}+16u_{i+1}-30u_i+16u_{i-1}-u_{i-2})$$

The truncation error here is

$$F_i = u_i' - \left(\frac{1}{30}\right)h^4\, u^{(5)}$$

and

$$S_i = u_i'' - \left(\frac{1}{90}\right)h^4\, u^{(6)}$$

Even though the new scheme and the standard representation both represent fourth order accuracy, the compact method should generate slightly more correct results due to smaller coefficients of the truncation error terms.

Difference scheme using compact scheme for and central difference scheme for and .

$$u_{tt}|_i^n = \frac{u_i^{n+1} - 2u_i^n + u_i^{n-1}}{k^2} + O(k^2)$$

and

$$u_t|_i^n = \frac{u_i^{n+1} - u_i^{n-1}}{2k} + O(k^2)$$

Then

$$u_{tt}|_i^n - u_{tt}|_{i-1}^n = \frac{(u_i^{n-1} - u_i^{n-1}) - 2(u_i^n - u_{i-1}^n) - (u_{i-1}^{n-1} + u_{i-1}^{n+1})}{k^2}$$

and

$$u_t|_i^n - u_t|_{i-1}^n = \frac{u_i^{n+1} - u_i^{n-1} - u_{i-1}^{n+1} + u_{i-1}^{n-1}}{2k}$$

Then by using the last results, we have from eqs. (19), (25), (24),(18), (26) and (27) as below:

$$u_0^n = 0$$

$$\left(\frac{\alpha}{k^2} + \frac{\beta}{2k}\right)u_1^{n+1} - \frac{6c^2}{h}F_0^n - \frac{6c^2}{h}F_1^n = \left(\frac{2\alpha}{k^2} - \frac{12c^2}{h^2} - \gamma\right)u_1^n + \left(\frac{\beta}{2k} - \frac{\alpha}{k^2}\right)u_1^{n-1}$$

$$F_{i-1}^n + 4F_i^n + F_{i+1}^n = \frac{3}{h}(u_{i+1}^n - u_{i-1}^n)$$

$$\left(\frac{\alpha}{k^2} + \frac{\beta}{2k}\right)u_i^{n+1} + \frac{c^2}{2h}F_{i+1}^n - \frac{c^2}{2h}F_{i-1}^n = \frac{2c^2}{h^2}(u_{i+1}^n + u_{i-1}^n) + \left(\frac{2\alpha}{k^2} - \frac{4c^2}{h^2} - \gamma\right)u_i^n + \left(\frac{\beta}{2k} - \frac{\alpha}{k^2}\right)u_i^{n-1}$$

$$u_m^n = 0$$

$$\left(\frac{\alpha}{k^2} + \frac{\beta}{2k}\right)u_{m-1}^{n+1} + \frac{6c^2}{h}F_{m-1}^n + \frac{6c^2}{h}F_m^n = \left(\frac{2\alpha}{k^2} - \frac{12c^2}{h^2} - \gamma\right)u_{m-1}^n + \left(\frac{\beta}{2k} - \frac{\alpha}{k^2}\right)u_{m-1}^{n-1}$$

Now for finding for the next time level, we use the initial condition

$u_t|_i^0 = g(x_i), \qquad 0 \le x \le l$

Which can be approximated into the form by using Taylor's series and finite differences as given in eq. (12). The Fourth Order Compact Scheme can be expressed in matrix form.

Application 1: Let us consider the homogeneous telegraph equation u_{t} in the interval . The boundary conditions are

$u(0,t) = u(\pi,t) = 0$

and the initial conditions are

and u,

$($

.

The Exact Solution is $u($.

Comparison of the Numerical Results of FDM and FOCM at

Table 1: Finite difference method

x_i	FDM	Exact	Error
0.000000000 0.000000000	0.000000000	0.000000000	
0.314159265	0.225789472	0.225706846	0.000065573
0.628318531	0.429476947	0.429319918	0.000157029
0.942477796	0.591124713	0.590908170	0.000216543
1.256637061	0.694908500	0.694654226	0.000254274
1.570796327	0.730607786	0.730402708	0.000267506
1.884955592	0.694908500	0.694654226	0.000254274
2.199114858	0.591124713	0.590908170	0.000216543
2.513274123	0.429476947	0.429319918	0.000157029
2.827433388	0.225789472	0.225706846	0.000082626
3.141592654	0.000000000	0.000000000	0.000000000

Table 2: Fourth order compact method

x_i	FOCM	Exact	Error
0.000000000	0.000000000	0.000000000	0.000000000
0.314159265	0.225772419	0.225706846	0.000065573
0.628318531	0.429440009	0.429319918	0.000120090
0.942477796	0.591074269	0.590908170	0.000166100
1.256637061	0.694849272	0.694654226	0.000195050
1.570796327	0.730607786	0.730402708	0.000205080
1.884955592	0.694849272	0.694654226	0.000195050
2.199114858	0.591074269	0.590908170	0.000166100
2.513274123	0.429440009	0.429319918	0.000120090
2.827433388	0.225772419	0.225706846	0.000065573
3.141592654	0.000000000	0.000000000	0.000000000

For graph see Figure 1

Application 2: Let us consider the homogeneous Telegraph equation in the interval . The boundary conditions are

$$u(0,t) = u(1,t) = 0$$

and the initial conditions are

and , .

The Exact Solution is .

Comparison of the Numerical Results of FDM and FOCM at t=0.05

Table 3: Finite difference method

x_i	FDM	Exact	Error
0.000000000	0.000000000	0.000000000	0.000000000
0.100000000	0.189019874	0.188654259	0.000365615
0.200000000	0.359537184	0.358841747	0.000695437
0.300000000	0.494860351	0.493903279	0.000957072
0.400000000	0.581743300	0.580618143	0.001125157
0.500000000	0.611681104	0.610498011	0.001183093
0.600000000	0.581743300	0.580618143	0.001125157
0.700000000	0.494860351	0.493903279	0.000957072
0.800000000	0.359537184	0.358841747	0.000695437
0.900000000	0.189019874	0.188654259	0.000365615
1.000000000	0.000000000	0.000000000	0.000000000

Table 4: Fourth order compact method

x_i	FOCM	Exact	Error
0.000000000	0.000000000	0.000000000	0.000000000
0.100000000	0.188897592	0.188654259	0.000243333
0.200000000	0.359286293	0.358841747	0.000444546
0.300000000	0.494860351	0.493903279	0.000610882
0.400000000	0.581743300	0.580618143	0.000718743
0.500000000	0.611681104	0.610498011	0.000755415
0.600000000	0.581743300	0.580618143	0.000718743
0.700000000	0.494860351	0.493903279	0.000610882
0.800000000	0.359286293	0.358841747	0.000444546
0.900000000	0.188897592	0.188654259	0.000243333
1.000000000	0.000000000	0.000000000	0.000000000

For graph see Figure 2

Application 3: Let us consider the homogeneous Telegraph equation in the interval

. The boundary conditions are

$$u(0,t) = u(\pi,t) = 0$$

and the initial conditions are

and , .

The Exact Solution is .

Comparison of the Numerical Results of FDM and FOCM at

Table 5: Finite difference method

x_i	FDM	Exact	Error
0.000000000	0.000000000	0.000000000	0.000000000
0.314159265	0.189019874	0.188654259	0.000365615
0.628318531	0.359537184	0.358841747	0.000695437
0.942477796	0.494860351	0.493903279	0.000957072
1.256637061	0.581743300	0.580618143	0.001125157
1.570796327	0.611681104	0.610498011	0.001183093
1.884955592	0.581743300	0.580618143	0.001125157
2.199114858	0.494860351	0.493903279	0.000957072
2.513274123	0.359537184	0.358841747	0.000695437
2.827433388	0.189019874	0.188654259	0.000365615
3.141592654	0.000000000	0.000000000	0.000000000

Table 6: FOURTH ORDER COMPACT METHOD

x_i	FOCM	Exact	Error
0.000000000	0.000000000	0.000000000	0.000000000
0.314159265	0.188897634	0.188654259	0.000243375
0.628318531	0.359286293	0.358841747	0.000444546
0.942477796	0.494514161	0.493903279	0.000610882
1.256637061	0.581336886	0.580618143	0.000718743
1.570796327	0.611253426	0.610498011	0.000755415
1.884955592	0.581336886	0.580618143	0.000718743
2.199114858	0.494514161	0.493903279	0.000610882
2.513274123	0.359286293	0.358841747	0.000444546
2.827433388	0.188897634	0.188654259	0.000243375
3.141592654	0.000000000	0.000000000	0.000000000

For graph see Figure 3

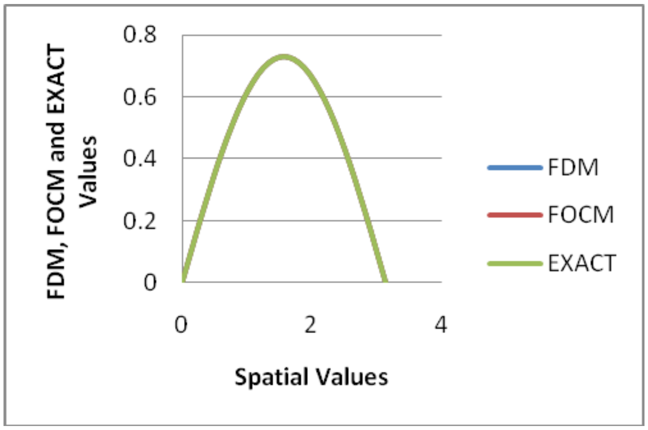

FIGURE 1. Comparison of FDM, FOCM and EXACT values at

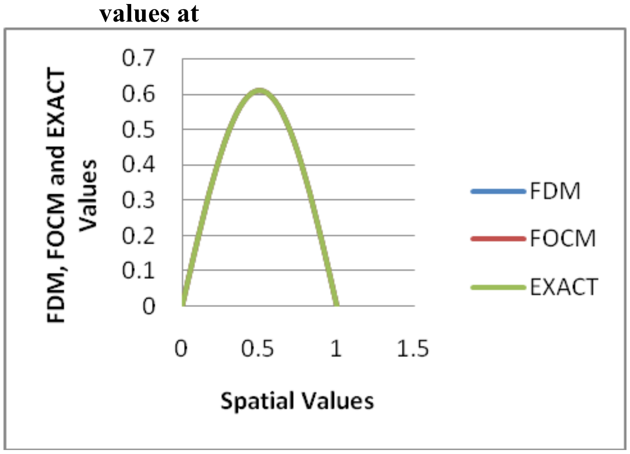

FIGURE 2. Comparison of FDM, FOCM and EXACT values at

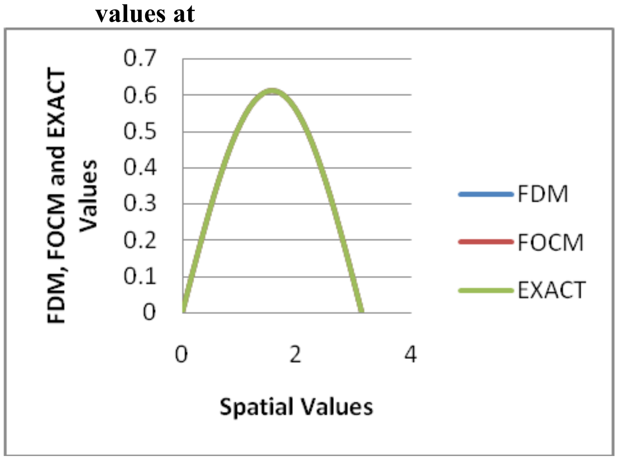

FIGURE 3. Comparison of FDM, FOCM and EXACT values at

RESULTS AND DISCUSSION

In this paper, numerical solutions for one dimensional linear homogeneous Telegraph equation are derived using Finite Difference Method (FDM) and Fourth Order Compact Method (FOCM). Fourth Order Compact Method is known to be a powerful method for solving such hyperbolic equations. Three different problems are tested numerically using finite difference method and fourth order compact method. The numerical results are then compared with the exact solutions. We observe that the fourth order compact method, which also uses only three nodes, gives better results than the standard finite difference second order method.

REFERENCES

Mohanty, R. K., An unconditionally stable difference scheme for one space dimensional linear Hyperbolic Equation, App. Math. Lett. 17,101-105(2004).

Orszag. S. A. and M. Israeli, Numerical Simulation of Viscous Incompressible Flows, Annual Review of Fluid Mechanics by Milton Van Dyke, Vol. 6, Annual Reviewers Inc., Palo Alto, California, 281-318(1974).

Ozair, M. A., An Exploration of Compact Finite Difference Methods For the Numerical Solution of PDE, Ph.D. Thesis, University of Western Ontario, Canada (1997).

Ozair, M. A., A Compact Method for the heat Equations, PJS, Vol. 58,106-110(2006).

Richard L. Burden, Numerical Analysis, 8th edition, Brooks Cole, (2004).

S.H. Leventhal, Melvin C., A note on the operator compact implicit method for the wave equation, Math Comput, 32(1), 143-147(1978).

Twizell, E. H., An Explicit difference method for the wave equation with extended stability range, BIT, 19, 378-383(1979).

Wazwaz, A. M., Partial Differential Equations and Solitary Waves Theory, Higher Education Press, Beijing, 388-394(2009).

Yiping Fu, Compact Fourth Order Finite Difference Schemes for Helmholtz Equation with High Wave Numbers, J. Comput. Math, Vol. 26, 98-111(2008).