CYCLICITY AND LITHOFACIES MODELLING BY APPLICATION OF MARKOV CHAIN ANALYSIS TO THE WARCHHA SANDSTONE, SALT RANGE, PAKISTAN

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ABSTRACT: The Early Permian Warchha Sandstone is well exposed in the Salt Range and dominantly composed of sandstone, siltstone and claystone sequence. Seven lithofacies have been identified in the Warchha Sandstone on the basis of geometry, gross lithology and sedimentary structures. Lithofacies are cyclically arranged in a fining-upward pattern. A complete cycle starts with pebbly sandstone accomplished by thin layer of basal conglomerate and terminates with claystone. The non-stationary first order Markov chains have been applied statistically on vertical transition of facies in outcrop data of the Warchha Sandstone succession from the Matan and the Sanwans areas of the Salt Range. A stimulation strategy was developed by transition intensity matrix to determine the facies-by-facies transitions. Later, a chi-square test was applied to test the dependency between any two facies states, during facies transitions. This study strongly support that the sediments of the Warchha Succession was deposited by Makovian mechanism and as a whole represents fluvial sedimentation deposited in a predictable cyclic arrangements of lithofacies.

Key words: Warchha Sandstone, lithofacies, first order Markov chains, Salt Range, Makovian mechanism, fluvial sedimentation.

INTRODUCTION

The Salt Range is 180 km long and 85 km wide range, located at the southern edge of the hydrocarbon bearing Potwar Basin (Fig.1). It presents a longitudinal east-west trending trough, bounded on east by the Jhelum River and on west by the Indus River, on the north by the Potwar Basin and on south by the Punjab Plain. The Early Permian rocks of the Salt Range belong to the northern Gondwana regimes and mainly comprise of siliciclastic sedimentary rocks, known as the Nilawahan Group (Ghazi, 2009). This group is classified in to four formations from base to top; the Tobra, Dandot, Warchha and Sardhai formations.

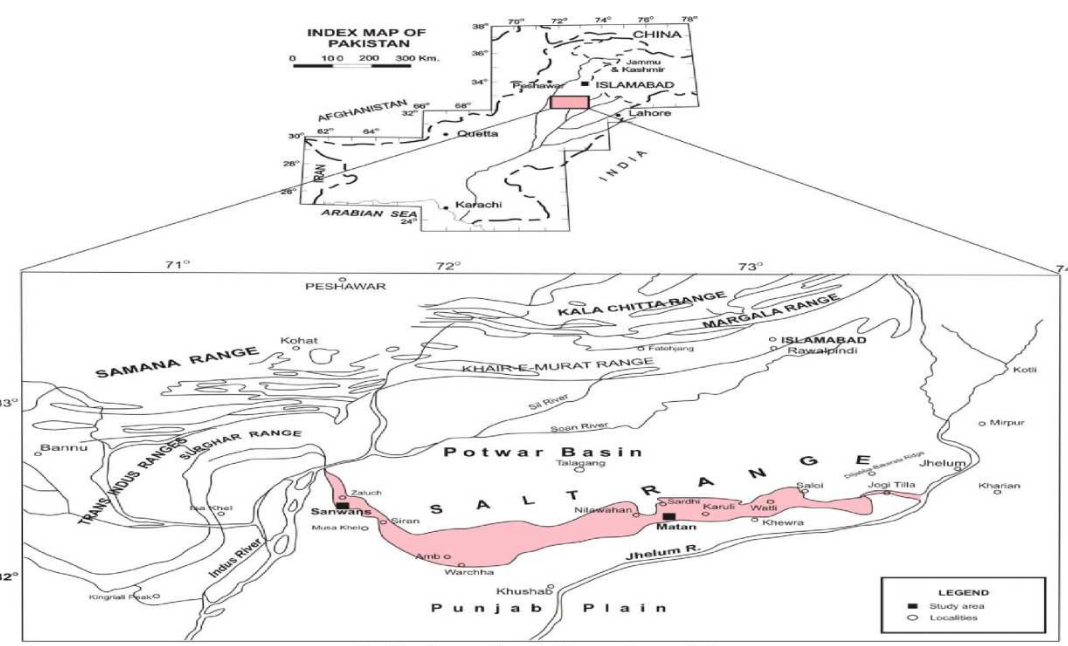

Fig. 1. Location map of the Matan and the Sanwans areas in the Salt Range, Pakistan.

The present study is focused on the outcrop data from the Matan central (Lat 32˚39′10″ N: Long 72˚36′30″ E) and the Sanwans (Lat 32˚42′57″ N: Long 71˚38′26″ E) western parts of the Salt Range (Fig. 1). Warchha Sandstone is well exposed throughout the Salt Range and its thickness increases from east to west. The detailed sedimentological study has indicated that it is a fining - upward, mainly continental basin-fill sequence (Ghazi, 2009). Cyclic characters of the Warchha Sandstone have been studied by applying Markov chain analysis (cf. Gingerich, 1969; Miall, 1973; Cant and Walker; 1976; Carr, 1982; Power and Easterling, 1982 and Harper, 1984). The aim of this paper is to recognize statistically various lithofacies arrangement, probability of transitions from one facies to another and to record possible large-scale depositional environments of the Warchha Sandstone.

Lithofacies analysis: The Warchha Sandstone can be informally divided into several conglomerate, sandstone and claystone units, with roughly equal proportions of sand and clay and rather less conglomerate (Ghazi, 2009). Conglomerate units are composed of rounded, sub-rounded and sub-angular clasts of igneous, metamorphic and sedimentary origin, which are dark-pink, dark-brown, maroon, white and green in colour. Clasts within the conglomerates mostly range from 0.5 cm to 2 cm in diameter, though large, rounded to sub-rounded, pink granite cobbles up to 20 cm are also present. The clasts lie in poorly sorted sand-, silt- and clay-grade matrix of varied composition. In places, intraformational clasts of claystone are also present. Arkose sandstone units are medium- to thick-bedded (50 cm to 90 cm), mainly light-brown to pinkish-white in colour, fine- to coarse-grained, poorly- to moderately-sorted, with grains that are sub-angular to sub-rounded. The sandstones commonly contain 1-3 cm-thick, dark-brown, grey and green coloured claystone layers. Additionally, gravel and pebble lags composed of pink granite are common. The sandstone units are locally speckled in appearance and, in places, contain carbonaceous material. Claystone units are red, maroon, dark-brown, grey and light-green in colour. They are commonly massive-bedded, blocky and splintery, though in places are interlaminated with thin, red, maroon, dark grey and dark green siltstone layers to form shales. A broad range of sedimentary structures is recognised, including different forms of bedding, cross-bedding, ripple marks, channels, flute casts, load casts, desiccation cracks, rain prints, cone-in-cone structures, a variety of concretions and bioturbation (Ghazi, 2009). The occurrence and abundance of these structures varies in a systematic manner throughout the vertical thickness of the succession.

Seven lithofacies, arranged into a series of discrete separate fining-upward cycles, are recognised in the Warchha Sandstone (Table 2; Fig. 2) and are here described based on the classification scheme of Miall (1985, 1996).

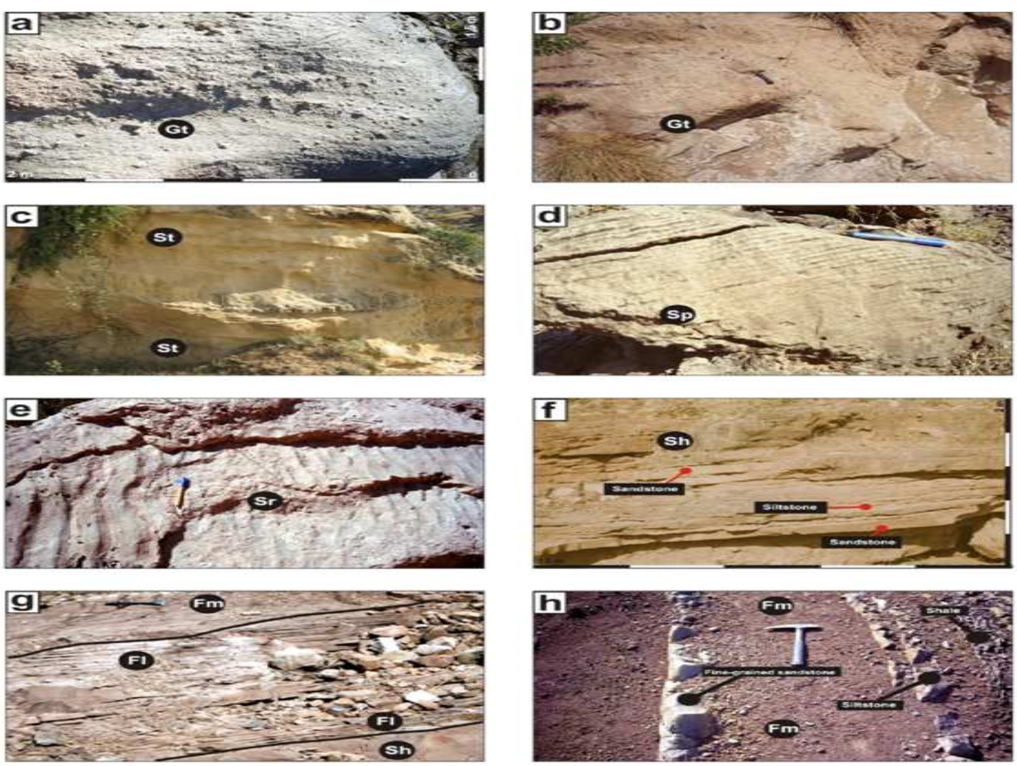

Fig. 2- Characteristic example of lithofacies in the Warchha Sandstone in the Matan and the Sanwans areas of the Salt Range.

A complete cycle of the lithofacies observed in surficial studies of the Warchha Sandstone was condensed into seven lithofacies states from bottom to top (Table 1).

Table 1. Summary of the characteristic features of the lithofacies types encountered in the outcrop sections of Warchha Sandstone from the Matan and the Sanwans area in the Salt Range, Pakistan. Seven facies are arranged in the predictable order to show a complete cycle of the Warchha Sandstone.

Facies	Characteristic features	Interpretation
Massive claystone / mudstone, Fm	This facies consists of red, dark-brown, green and yellow claystone and shale with occasional grey to greenish-grey siltstone interbeds. The facies is generally massive, though at a few horizons it contains abundant bioturbation, clay balls, iron concretions, desiccation cracks, rain-drop imprints and caliche nodules up to 10 cm in diameter. The lower contact of this facies is typically gradational, whereas the upper contact is usually sharply truncated by the erosive base of the overlying cycle.	This facies is interpreted to represent deposition from suspension in overbank settings where the fine-grained sediments drape underlying deposits.
Parallel laminated siltstone and claystone, Fl	This facies consists of laminated siltstone and/or massive claystone. Its lower with facies Sh or Sr and upper contact with facies Fm are gradational. Common structures include clay balls and iron concretions. Interlaminated siltstone horizons exhibit very small ripple marks and lenticular bedding. Geometrically, this facies is arranged into thin but laterally extensive sheet-like bodies.	This facies is interpreted to represent the deposits of waning stage flood deposition, chiefly in overbank areas, with the majority of deposition occurring from suspension settling and with only limited bedload transport via weak currents.
Very fine- to medium-grained sandstone with flat bedding, Sh	This facies consists of very fine- to medium-grained, horizontally laminated sandstone arranged into thin beds with a sheet or tabular geometry.	Deposited as a plane bed under conditions of either upper or lower flow regime, either on bar top surfaces or as isolated sand sheets in overbank flood plain areas.
Ripple cross-laminated sandstone, Sr	This facies overlies facies Sp and consists of fine- to coarse-grained sandstone, which is interlaminated with thin siltstone and claystone horizons. The sandstone is medium- to thick-bedded. It occurs as thin wedge-shaped bodies which pinch out laterally within few metres and which contain abundant ripple marks, flat bedding, and small-scale trough and planar cross-stratification and load casts. Clay intraclasts and concretions are common in the upper parts of the deposits, as are sand balls with diameters of 50-80 cm, which are a particularly diagnostic feature.	This facies likely represents the temporary abandonment of bars during periods of elevated water level and/or the product of deposition in areas of slack or sluggish water between bars or in overbank areas.
Medium- to coarse-grained planar cross-bedded sandstone, Sp	This facies consists of medium- to coarse-grained, poorly sorted, arkosic sandstone arranged into lenticular or tabular sets, which are characterised internally by planar cross-bedding. The lower contact of this facies is sharp and flat, whereas the upper contact is erosional either with facies Sr or Fl.	Deposited as dunes or bars under conditions of lower flow regime.
Coarse-grained trough cross-bedded sandstone, St	This facies is most commonly overlies facies Gt and consists of medium- to very coarse-grained sandstone arranged into trough cross-bedded sets and cosets. Trough cross-bedded sets are typically 0.3-0.5 m thick and individual troughs are up to 1 m wide. The inclination of larger foresets varies from 8-16°. Geometrically, this facies occurs as lenticular or wedge-shaped bodies that are pebbly in places and which are commonly arranged into stacked trough cross-bedded cosets. The lower boundary is either gradational with facies Gt or is erosional with facies Fm, whereas the upper contact is sharp and flat with facies Sp.	Deposited as dunes or bars under conditions of lower flow regime.
Stratified gravely sandstone, Gt	This facies is always present as the lowermost deposits at the base of each complete cycle. It consists of trough cross-bedded, stratified gravels that commonly infill channel-like erosive basal surfaces. Geometrically, the facies consists of lens- or ribbon-shaped bodies, commonly interbedded with sandy deposits.	Deposited as channel lag under conditions of lower flow regime, with sediment transport occurring via traction currents.

Each complete cycle starts with a conglomerate or coarse-grained gravelly sandstone (facies Gt or St) at the base (Fig. 2a and b), succeeded in turn, by coarse to fine grained sandstones (facies Sp, Sh, Sr; Figs. 2b-e), parallel laminated siltstone-claystone (facies Fl) (Fig. 2f) and terminates with a claystone/mudstone intercalated with carbonaceous shale (facies Fm) (Fig. 2g). Sedimentation took place in a cyclic manner and associated sedimentary characteristics suggest that, as the deposition of each cycle proceeded, the flow intensity progressively decreased and was considerably reduced during the deposition of mudstone units that dominate the upper parts of each cycle (Ghazi, 2009; Fig. 3).

Markov chain analysis: Markov chain analysis is applied in geology to model discrete variables such as lithologies or facies (Krumbein and Dacey 1969; Kulatilake, 1987; Hota et al., 2003; Hota and Maejima, 2004; Ghazi, 2009). This analysis quantifies the geologic interpretation of juxtaposition tendencies of facies in vertical stratigraphic successions (Krumbein and Dacey 1969; Kulatilake). It also addresses the comparison between lateral juxtapositional tendencies of facies to those in vertical (i.e., Walther‟s Law). Markov chain analysis is based on the simple theory of whether a given lithology is independent or not on the lithology lying immediately below ((Miall, 1973; Kulatilake, 1987). The greater the dependence, the more likely that transition from one bed to another is part of a pattern of behaviour. The Markov chain model will be first order if transition of facies depends only on previous facies and higher order if transition of facies depends on more than one previous facies (Kulatilake, 1987).

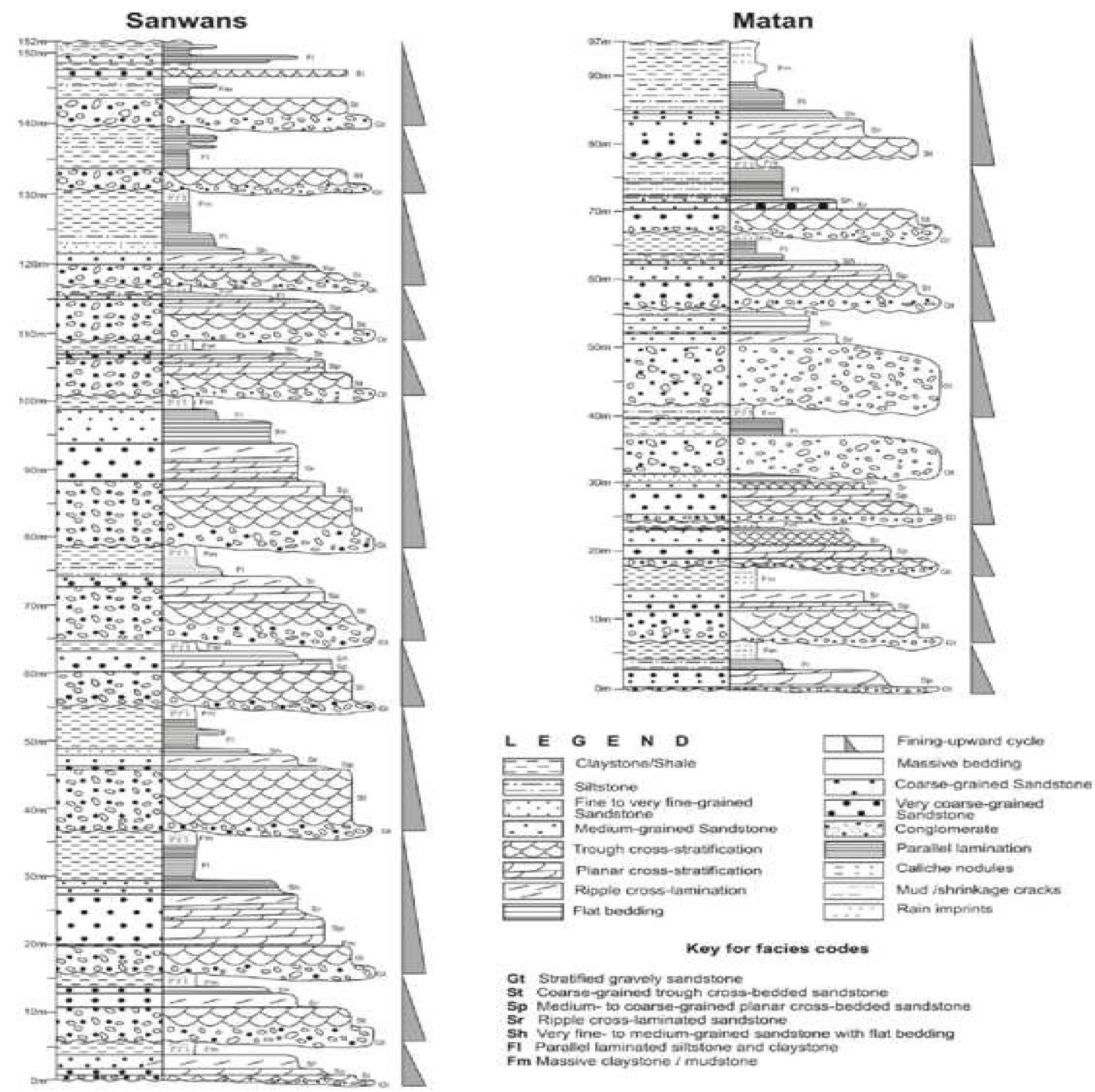

Fig. 3- Vertical arrangement of lithofacies in fining-upward cycle in the Warchha Sandstone from the Matan Central and the Sanwans Western parts of the Salt Range, Pakistan.

A Markov chain will be stationary if transition of facies does not changed, otherwise will be non-stationary or homogeneous (Kulatilake, 1987). This model compares the observed frequencies of facies transitions with the expected frequencies if all transitions were random. When expected frequencies are subtracted from observed frequencies, significantly positive differences are used to suggest an idealized succession of facies.

METHODOLOGY

The non-stationary or homogeneous first order Markov chains have been applied on vertical transition of facies data from the Matan and the Sanwans outcrop sections in the Warchha Sandstone. In the present study the process outlined by Walker (1984) and Harper (1984) is followed. Markov chain analysis of the Warchha Sandstone is based on the assumption that vertically the lithofacies at a point n depends upon the lithofacies at the proceeding point i.e. n-1 (cf. Kulatilake, 1987). The data were collated and processed into transition count, transition probability, random probability and difference matrices (Tables 2 and 3).

Table 2-(a-d)- Matrices used to analysed transitions of facies in the Warchha Sandstone in Matan area. (e) Binomial Probability (BP) for facies transitions I nthe Warchha Sandstone in Matan Area.

Table 2a - Transition count matrix

	Gt	St	Sp	Sr	Sh	Fl	Fm	Row Total (N)
Gt	0	5	1	1	0	1	0	8
St	0	0	4	2	0	0	0	6
Sp	0	0	0	3	1	1	0	5
Sr	0	0	0	0	5	0	1	6
Sh	1	0	0	0	0	3	2	6
Fl	0	0	0	0	0	0	5	5
Fm	6	1	0	0	0	0	0	7
Column Total	7	6	5	6	6	5	8	Grand Total = 43

Table 2b - Transition probability matrix

	Gt	St	Sp	Sr	Sh	Fl	Fm
Gt	0.000	0.625	0.125	0.125	0.000	0.125	0.000
St	0.000	0.000	0.670	0.330	0.000	0.000	0.000
Sp	0.000	0.000	0.000	0.600	0.200	0.200	0.000
Sr	0.000	0.000	0.000	0.000	0.830	0.000	0.167
Sh	0.167	0.000	0.000	0.000	0.000	0.500	0.330
Fl	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Fm	0.860	0.143	0.000	0.000	0.000	0.000	0.000

Table 2c - Random probability matrix

	Gt	St	Sp	Sr	Sh	Fl	Fm
Gt	0.000	0.167	0.139	0.167	0.167	0.139	0.220
St	0.190	0.000	0.135	0.160	0.160	0.135	0.217
Sp	0.180	0.158	0.000	0.158	0.158	0.131	0.210
Sr	0.190	0.162	0.135	0.000	0.160	0.135	0.217
Sh	0.100	0.162	0.135	0.162	0.000	0.135	0.217
Fl	0.184	0.158	0.131	0.158	0.158	0.000	0.210
Fm	0.200	0.170	0.143	0.170	0.170	0.143	0.000

Table 2d - Difference matrix

	Gt	St	Sp	Sr	Sh	Fl	Fm
Gt	0.000	+0.458	-0.010	-0.042	-0.167	-0.010	-0.220
St	-0.190	0.000	+0.535	+0.170	-0.160	-0.135	-0.217
Sp	-0.180	-0.158	0.000	+0.440	+0.042	+0.068	-0.210
Sr	-0.190	-0.162	-0.135	0.000	+0.670	+0.135	-0.050
Sh	+0.157	-0.162	-0.135	-0.162	0.000	+0.365	+0.113
Fl	-0.184	-0.158	-0.131	-0.1404	-0.158	-0.158	+0.790
Fm	+0.660	-0.020	-0.143	-0.170	-0.170	-0.143	0.000

Table 2e -Binomial probability

Facies transition	P	N	n	Probability
Gt to St	0.167	8	5	0.0085
St to Fm	0.220	6	0	0.9900
Sp to Sr	0.160	5	3	0.0031
Sp to St	0.160	5	0	1.0000
Sr to Sh	0.160	6	5	0.0005
Sh to Fl	0.135	6	3	0.0370
Fl to Fm	0.210	5	5	0.0004
Fm to Gt	0.200	7	6	0.0004
Fm to St	0.170	7	1	0.7200

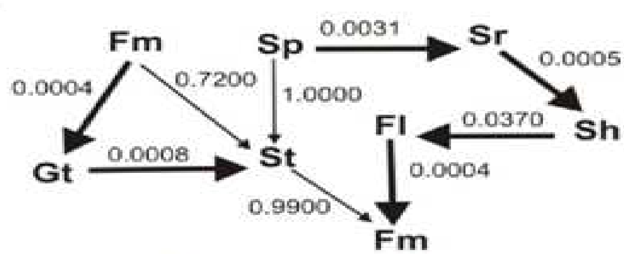

Fig. 4- Facies Relationship Diagram (FRD) based on positive values of the difference matrix recorded in Table 2d, showing frequently upward transition of facies states in clearly defined cycles in the Warchha Sandstone in the Matan area. Numbers showing probabilities of passing upward from one lithofacies to another. For detail explanation see the text.

Table 3-(a-d)- Matrices used to analysed transitions of facies in the Warchha Sandstone in Matan area. (e) Binomial Probability (BP) for facies transitions in the Warchha Sandstone in the Sanwans area, Salt Range.

Markov chain analysis of the Sanwans area, Salt Range

Table 3a - Transition count matrix

	Gt	St	Sp	Sr	Sh	Fl	Fm	Row Total (N)
Gt	0	12	0	0	0	0	0	12
St	0	0	8	1	0	3	1	13
Sp	0	0	0	7	1	1	0	9
Sr	0	0	0	0	6	1	1	8
Sh	1	0	0	0	0	4	3	7
Fl	1	1	0	0	0	0	6	8
Fm	10	0	1	0	0	0	0	11
Column Total	11	13	9	8	7	9	11	Grand Total = (

Table 3b - Transition probability matrix

	Gt	St	Sp	Sr	Sh	Fl	Fm
Gt	0.000	1.000	0.000	0.000	0.000	0.000	0.000
St	0.000	0.000	0.615	0.077	0.000	0.230	0.077
Sp	0.000	0.000	0.000	0.780	0.110	0.110	0.000
Sr	0.000	0.000	0.000	0.000	0.750	0.125	0.125
Sh	0.000	0.000	0.000	0.000	0.000	0.570	0.430
Fl	0.000	0.000	0.000	0.000	0.000	0.000	1.000
Fm	0.860	0.143	0.000	0.000	0.000	0.000	0.000

Table 3c - Random probability matrix

	Gt	St	Sp	Sr	Sh	Fl	Fm
Gt	0.000	0.230	0.158	0.140	0.123	0.158	0.192
St	0.200	0.000	0.164	0.145	0.127	0.163	0.200
Sp	0.186	0.220	0.000	0.135	0.120	0.152	0.187
Sr	0.183	0.217	0.150	0.000	0.116	0.150	0.183
Sh	0.180	0.210	0.148	0.130	0.000	0.148	0.180
Fl	0.187	0.220	0.150	0.135	0.120	0.000	0.190
Fm	0.190	0.230	0.158	0.140	0.123	0.158	0.000

Table 3d - Difference matrix

	Gt	St	Sp	Sr	Sh	Fl	Fm
Gt	0.000	+0.770	-0.158	-0.140	-0.123	-0.158	-0.192
St	-0.200	0.000	+0.451	-0.068	-0.127	+0.067	-0.123
Sp	-0.186	-0.220	0.000	+0.645	-0.009	-0.042	-0.187
Sr	-0.183	-0.217	-0.150	0.000	+0.634	-0.025	-0.058
Sh	-0.180	-0.210	-0.148	-0.130	0.000	+0.422	+0.250
Fl	-0.062	-0.095	-0.150	-0.135	-0.120	0.000	+0.560
Fm	+0.720	-0.230	-0.068	-0.140	-0.123	-0.158	0.000

Table 3e -Binomial probability

Facies transition	P	N	n	Probabilit
Gt to St	0.230	12	12	$2.2x10^{-8}$
St to Sp	0.163	13	8	0.0003
St to Fl	0.163	13	3	0.5700
Sp to Sr	0.135	9	7	0.0002
Sp to Sh	0.120	9	1	0.6800
Sr to Sh	0.116	8	6	0.00005
Sh to Fl	0.148	7	4	0.0110
Sh to Fm	0.180	7	3	0.1100
Fl to Fm	0.190	8	6	0.0009
Fm to Gt	0.190	11	10	$5.5x10^{-7}$

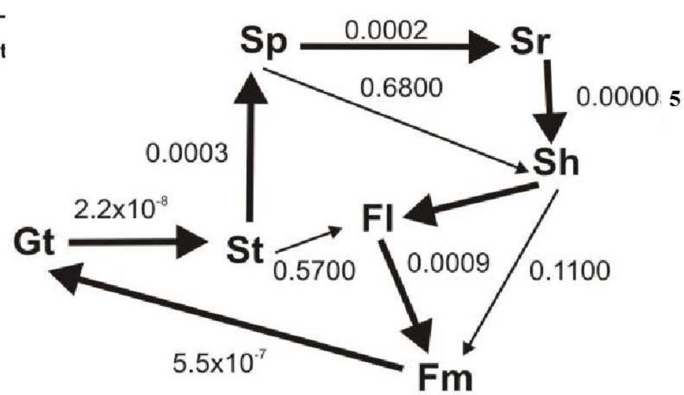

Fig. 5- Facies Relationship Diagram (FRD) based on positive values of the difference matrix recorded in Table 2d, showing frequently upward transition of facies states in clearly defined cycles in the Warchha Sandstone in the Sanwanarea. Numbers showing probabilities of passing upward from one lithofacies to another. For detail explanation see the text.

On the basis of the difference matrix, results in the form of facies relation diagrams were prepared to show the transition of facies states from one type to another (Figs. 4 and 5). The cyclicity of facies was tested using a standard chi-square test (Tables 4 and 5).

Table 4 & 5- Chi-square statistical analysis of the Warchha Sandstone from the Matan and the Sanwans areas, Salt Range.

Chi-square statistics of the Matan area, Salt Range

Table 4a - Observed frequencies (O)

	Gt	St	Sp	Sr	Sh	Fl	Fm	Row Total (N)
Gt	0	5	1	1	0	1	0	8
St	0	0	4	2	0	0	0	6
Sp	0	0	0	3	1	1	0	5
Sr	0	0	0	0	5	0	1	6
Sh	1	0	0	0	0	3	2	6
Fl	0	0	0	0	0	0	5	5
Fm	6	1	0	0	0	0	0	7
Column Total	7	6	5	6	6	5	8	Grand Total = 43

Table 4b - Expected frequencies (E)

	Gt	St	Sp	Sr	Sh	Fl	Fm
Gt	1.30	1.12	0.93	1.12	1.12	0.93	1.49
St	0.98	0.84	0.70	0.84	0.84	0.70	1.12
Sp	0.82	0.70	0.58	0.70	0.70	0.58	0.93
Sr	0.98	0.84	0.70	0.84	0.84	0.70	1.12
Sh	0.98	0.84	0.70	0.84	0.84	0.70	1.12
Fl	0.82	0.70	0.58	0.70	0.70	0.58	0.93
Fm	1.14	0.98	0.81	0.98	0.98	0.81	1.30

Table 4c- Chi-square values $(O-E)^2/E$

	Gt	St	Sp	Sr	Sh	Fl	Fm
Gt	1.30	13.44	0.01	0.01	1.12	0.01	1.49
St	0.98	0.84	15.68	1.60	0.84	0.70	1.12
Sp	0.82	0.70	0.58	7.56	0.13	0.30	0.93
Sr	0.98	0.84	0.70	0.84	20.60	0.70	0.01
Sh	0.00	0.84	0.70	0.84	0.84	7.63	0.70
Fl	0.82	0.70	0.58	0.70	0.70	0.58	17.81
Fm	20.58	0.00	0.81	0.98	0.98	0.817	1.30
Sum of X^2	25.58	1736	19.05	12.53	25.21	10.72	23.36

Grand Sum of X^2 = 133.82

Chi-square statistics of the Sanwans area, Salt Range

Table 5a - Observed frequencies (O)

	Gt	St	Sp	Sr	Sh	Fl	Fm	Row Total (N)
Gt	0	12	0	0	0	0	0	12
St	0	0	8	1	0	3	1	13
Sp	0	0	0	7	1	1	0	9
Sr	0	0	0	0	6	1	1	8
Sh	0	0	0	0	0	4	3	7
Fl	1	1	0	0	0	0	6	8
Fm	10	0	1	0	0	0	0	11
Column Total	7	6	5	6	6	5	8	Grand Total = 68

Table 5b - Expected frequencies (E)

	Gt	St	Sp	Sr	Sh	Fl	Fm
Gt	1.94	2.28	1.56	1.44	1.24	1.56	1.94
St	2.11	2.47	1.69	1.56	1.34	1.69	2.11
Sp	1.46	1.71	1.17	1.08	0.93	1.17	1.46
Sr	1.30	1.52	1.04	0.96	0.82	1.04	1.30
Sh	1.13	1.33	0.91	0.84	0.72	0.91	1.13
Fl	1.30	1.52	1.04	0.96	0.82	1.04	1.30
Fm	1.78	2.09	1.43	1.32	1.13	1.43	1.78

Table 5c- Chi-square values $(O-E)^2/E$

	Gt	St	Sp	Sr	Sh	Fl	Fm
Gt	1.94	44.44	1.56	1.44	1.24	1.56	1.94
St	2.11	2.47	23.56	0.20	1.34	1.02	0.58
Sp	1.46	1.71	1.17	32.45	0.01	0.02	1.46
Sr	1.30	1.52	1.04	0.96	32.51	0.00	0.07
Sh	1.13	1.13	0.91	0.84	0.72	10.49	3.07
Fl	0.07	0.18	1.04	0.96	0.82	1.04	17.07
Fm	37.90	2.09	0.13	1.32	1.13	1.43	1.78
Sum of X^2	45.90	50.74	29.41	38.17	37.77	15.56	25.98

Grand Sum of X^2 = 243.53

Transition count matrix: A vertical sequence of facies was coded using letters and symbols to facilitate tabulation of transition from one facies to another. The coded sequence is read like words in paragraph. Starting with facies Gt (Tables 2 and 3), the sequence is read as "facies Gt overlain by a scoured surface overlain by facies St overlain by facies Sr, etc." The observed number of transitions were tabulated and put into a Tally Matrix (Tables 2a and 3a). The data of this matrix shows that facies Gt passes upward into facies St five times. Row totals of the tally matrix indicate the total number of transitions in each row. For instance, facies Gt passes upward into another facies eight times (Table 2a). Column totals in the tally matrix indicate the total number of transitions in each column. For example, different facies pass upward into the facies Gt seven times (Table 2a).

Transition probability matrix

A second matrix was constructed by converting the observed number of transitions to transition probabilities (Tables 2b and 3b), for vertical sampling interval. This was done by dividing the number of transitions in each cell by the row total. The estimated transition probability matrix was found to be sensitive to the sampling interval used.

Random probability matrix: An independent trials or random probability matrix was constructed (Tables 2c and 3c), which is based on the assumption that all facies transitions are random. The value in each cell of this matrix was calculated by using the Eq. 1.

$$R_{ij} = SC_j / (T - SC_i) \qquad (Eq. 1)$$

Where

R_{ij} = Random Probability of transition from facies i to j.

SC_j = Random number of occurrences of facies j (i.e. column total for facies j).
T = Total number of transitions for all facies.
SC_i = Number of occurrences of facies i.

In the random probability matrix, all the diagonal cells are "structurally empty", (i.e. all values = 0). This is the characteristic feature of an embedded Markov chain (Miall, 1973; Kulatilake, 1987). t occurs because of the impossibility of recognizing the transition of a given facies upward into another bed of the same facies.

Difference matrix: A Difference matrix (Tables 2d and 3d) was constructed showing the observed minus the random probabilities. This was done by subtracting the value of each cell in tables 2c and 33, from the corresponding cell in tables 2b and 3b. The difference matrix values of the ranges from +1 to –1. Positive values indicate transitions that occur more frequently than would be the case if the facies were arranged randomly. Whereas, the negative values indicate transitions of facies occur less frequently. A facies relationship diagrams (FRD) were drawn using those transitions with "high" positive values (Figs. 4 and 5). A positive value in the difference matrix doesn‟t prove that the difference is statistically significant. Indeed, one of the most serious problems with the method described above is the uncertainty of determining which transitions are significant (cf. Cant and Walker, 1976). To remedy this difficulty, Harper (1984) suggested the use of binomial probability of at least N_{obs} *successes* in N trials, given by Eq. 2.

$$\sum_{n=nobs}^{n=N} = C\ (N, n)\ p^n\ \ q^{N-n} \qquad (Eq.\ 2)$$

Where
C (N, n) = the number of possible combinations of N objects taken n at a time, and is given by: C (N,n) = N! / (N-n)! n!
q = 1- p
N = Total number of upward transitions of any facies into all other facies (i.e. the row totals in tables 2a and 3a)
n = Number of upward transitions of any facies into any other facies (e.g. in case of Gt to St, n = 5 in table 2a)
p = Probability of success on a single trial
q = Probability of failure on a single trial

Simulation of binomial probability method: A null hypothesis was set up, which states: All the transitions of any facies into another facies occur randomly in the Warchha Sandstone. The independent N trials are established at significance level of 0.1 (i.e. 10%). A criterion is set up that to accept or reject the null hypothesis if the value of computed probability is greater than or equal to the level of significance chosen- it will be rejected. Otherwise, the null hypothesis would be accepted. The significance level 0.10 represents a particular Binomial Probability (BP) value that would lead to a wrong decision regarding rejection or acceptance of the null hypothesis could occur by chance 10 times out of 100. The transition of facies Gt to St was considered to illustrate the binomial probability method. The tally matrix shows that facies Gt was converted upward into the facies St five times out of eight transitions from facies Gt upward into all facies: hence n_{obs} = 5 and N = 8 (Table 2a),. The binomial probability of at least n_{obs} successes in N trials is 0.0085 (Table 3a). This means that five successes in eight trials would occur 0.85% of the time, if the transitions occurred randomly. At a significance level of 0.1, the observed transition of facies Gt to St is "significant" or the transition is rare (i.e. less than 10% of the time, if the transitions occur randomly).

Chi-Square test: A chi-square test is applied to test the dependency between any two facies states, during facies transitions in the Warchha Sandstone. It also investigates the transition probability matrix (Tables 2b and 3b) as a whole for non-randomness, in facies modelling by Markov chain method.
A null hypothesis (H_0) was set up, which states "there is no difference between any two facies states, i.e. they are derived from the same bed". The "no difference" aspect is a structural feature of the null hypothesis. The observed frequencies were put in tabular form (Tables 4a and 5a). The expected frequency (Tables 4b and 5b) was calculated using the Eq 3.

$$E = (\ n_R \times n_c / N) \qquad (Eq.\ 3)$$

Where:
n_R = Row totals in tables, n_C = Column totals in tables, and
N = Grand total of facies transitions in tables
Some statistical references state that a chi-square test is valid only if none of the cell expected frequency values is less than five. However recent studies (Miller, 1983, Feinberg, 1980) show that this requirement may be relaxed so that several expected cell frequencies may in fact be less than one. So this criterion was ignored for the validity of the chi-square test, in present study.
The chi square values for each of the cell was calculated by using Eq. 4.

$$Chi-square = \chi^2(Observed\ value - Expected\ \ value)/\ Expected\ \ value \qquad (Eq.4)$$

Summation of the chi-square contributions gave chi-square values of 133.82 and 243.52, respectively (Tables 4c and 5c). The larger these values, the greater the difference between any two facies states of the Warchha Sandstone. But it is yet to be determined whether 133.82 and 243.52 are large enough to allow the rejection of the null hypothesis. So the degree of freedom was determined, before picking a significance level, and then the critical value for chi-square (X^2) at that significance level was calculated. The degree of freedom (d.f) for the N by N facies transition matrix was calculated by using Eq. 5.

$$d.f. = (N-1)^2 - N \qquad (Eq. 5)$$

Where
N = Total number of facies states in the Warchha Sandstone.

Thus for the Warchha Sandstone outcrop data, there are 29 degrees of freedom. A critical value for Chi-square was determined, which allowed us to either reject or accept the null hypothesis. So the rejection or acceptance of null hypothesis depends on the d.f. and significance level. A significance level of 0.05 (i.e. 5%) was picked up for the facies states of the Warchha Sandstone. Since the critical value for 29 degrees of freedom and a significance level of 0.05 is 42.56. Therefore, the computed value of Chi-square exceeds the critical value at the 5 % significance level (Tables 4c and 4c), suggesting the markovity and cyclic arrangement or non-stationarity of facies transition through time in the Warchha Sandstone.

DISCUSSION

The individual chi-square contributions of all the facies states in the Warchha Sandstone from the Matan and the Sanwans areas of the Salt Range show a variable degree of dependency on precursor and influence on successor states during the sedimentation of these cycles (Tables 4 and 5). The facies relationship diagrams (FRD) drawn up from the difference matrices also show that the cycles are fining-upward asymmetric type (Figs. 4 and 5). Each complete cycle starts with a conglomerate or coarse-grained gravelly sandstone (facies Gt or St) at the base, succeeded in turn, by coarse to fine grained sandstones (facies Sp, Sh, Sr), parallel laminated siltstone-claystone (facies Fl) and terminates with a claystone/mudstone intercalated with carbonaceous shale (facies Fm). The interbedded siltstone-claystone-carbonaceous shale sequence (facies Fl and Fm) at the top of the Warchha Sandstone, show a marked deviation from other states, which is due to relatively smaller values of chi square for these facies (cf. Hota and Maejima, 2004). The low values of the facies Fl and Fm indicate presence of variable depositional environments for „mudstone-carbonaceous shale succession in the Warchha Sandstone in overbank setting (Ghazi, 2009; Hota and Maejima, 2004). The alluvial floodplains may develop either locally or occupy the entire basin overlying the deposits of various sub environments (Strahaler, 1963; Hota and Maejima, 2004 Ghazi, 2009). Thus the facies states Fl and Fm may be laterally restricted or extensive and show different types of floor rocks as observed in the Tobra and Dandot formations throughout the central and western parts of the Salt Range (Ghazi, 2009).

Conclusions: The Early Permian Warchha Sandstone of the Matan from central and Sanwans from western parts of the Salt Range exhibits cyclic arrangement of lithofacies. Each complete cycle of the Warchha Sandstone starts with a conglomerate or coarse-grained gravelly sandstone (facies Gt or St) at the base, succeeded in turn, by coarse to fine grained sandstones (facies Sp, Sh, Sr), parallel laminated siltstone-claystone (facies Fl) and terminates with a claystone/mudstone intercalated with carbonaceous shale (facies Fm). These cycles are comparable to each other at various localities in the Salt Range and show fining-upward character, which might have resulted from variations in hydraulic changes of lateral shifting of the meander belts. Repeated occurrence of the sandstone-siltstone-claystone facies suggests that channel establishment and abandonment was repeated many times at a given site upward facies transition is non-stationary over the entire areas. Relatively low chi-square values for facies Fl and Fm indicate a broad regional variation in the depositional environments that are not significant at the local scale. The siltstone-claystone-carbonaceous-shale develop either locally in a part of the floodplain or occupy the entire basin overlying the deposits of various sub environments.

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