# RATIO ESTIMATORS FOR POPULATION VARIANCE IN ADAPTIVE CLUSTER SAMPLING

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**ABSTRACT:** To estimate the variance for rare and cluster population has been the main problem in survey sampling. Three ratio type estimators were proposed for population variance utilizing the single auxiliary variable assuming the transformed population for adaptive cluster sampling, in presented study. The expressions for the mean square error and bias of the proposed estimator were derived. The proposed estimatorswere used to estimate the finite population variance in adaptive cluster sampling. The simulationswereperformedon a real life data to reveal and evaluate the efficiency of the estimators. The results showed that the proposed exponential ratio estimator was more efficient compared to the usual sample variance estimator and the proposed ratio type variance estimators in adaptive cluster sampling, assuming given conditions. Hence, exponential ratio estimators were recommended to estimate the population variance in adaptive cluster sampling.

**Keywords:** Exponential Ratio Estimator, Transformed Population,Mean Squared Error,Bias,Auxiliary Variable.

## INTRODUCTION

Adaptive cluster sampling (ACS) is useful to measure the density of rareclustered population in survey sampling. Examples of these populations can be derived in mineral analysis, animal and plant populations of rare and endangered species, toxic wasteabsorption, epidemiology of syndromeand loudtroublesetc(Thompson, 1992). ACS has the broaduse in diversefieldsincluding,Biological (Lo *et al*., 1997 and Acharya *et al*., 2000), Ecological (Correll, 2001), Environmental (Vasudevan*et al*., 2001 and Smith *et al.*, 2003), Geological (Boomer *et al*.,2000), and Social Sciences (Thompson and Collins, 2002).

A conventional sampling design is used to select initial sample in ACS. A condition C is fixed to take account of anelement in the sample. Everyunit in the neighborhood is included and examined. If the fixed condition is fulfilledinongoingprocedure until new element satisfied the fixed condition. ACS is a kind of network sampling, which givesbetterapproximationincontrast to usualsampling designs. In case of scarceand group population, this type of network sampling is better than conventional design. The entireunits studied (together with the initial sample) arecomposed the ultimate sample. The group of elements which fulfill the fixed condition is recognized as network. The elementswhich do not fulfill the condition are identified as edge units. Cluster is a blend of network elements with associated edge elements (Thompson, 1992).A variety of estimators are available in conventional sampling design to estimate the population variance, but these estimators have low efficiency for rare and clustered population. There are no estimators available that utilizes the auxiliary variable to estimate population variance of the rare and clustered population. Thus, there is a need to deal with the efficiency issues and proposed better estimators appropriate for the population variance in adaptive design.

## MATERIALS AND METHODS

**Notations and Some Estimators in Conventional Sampling:**Supposed a random sample of size n was selected with simple random sampling without replacementfrom entire units N in the population. The variable of interest and auxiliary variable wererepresentedthrough y and x with the population means $\overline{Y}$ and $\overline{X}$, population standard deviations $S_y$ and $S_x$, coefficient of variations $C_y$ and $C_x$ respectively. It was assumed also that $\rho_{xy}$ symbolized thepopulation correlation coefficient between x and y. The sample means of the variable of interest and auxiliary variable wererepresentedwith $\overline{y}$ and $\overline{x}$. Let a finite population $U = (U_1, U_2, U_3,...,U_n)$ of dissimilar and exclusiveelements. Consider Y be a variable of interest with value $Y_i$ measured on $U_{i,} i = 1, 2, 3,...,n$ providing a vector of values $Y = (Y_1, Y_2, Y_3,...,Y_n)$. The

objectivewas to approximate the variance of population

$S_y^2 = \frac{1}{N-1}\left[\sum_{i=1}^{N}(y_i - \overline{Y})^2\right]$ with the source of a random sample of sizen, drawn from the population. Some notations to be used were described below:

$\gamma = \frac{1-f}{n}$, $\mu_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \overline{Y})^r(x_i - \overline{X})^s$

$\lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}$ and $\lambda_{22} = \frac{\mu_{22}}{\mu_{20}\mu_{02}}$

$\beta_{2(y)} = \frac{\mu_{40}}{\mu_{20}^2}$ and $\beta_{2(x)} = \frac{\mu_{04}}{\mu_{02}^2}$

Where $\lambda_{22}$ represent population correlation coefficients between $y$ and $x$ ,Where $\lambda_{40} = \beta_{2(y)}$ and $\lambda_{04} = \beta_{2(x)}$ were the kurtosis for the population of the study variable and the auxiliary variable respectively.

The usual sample variance estimator of the population variance was defined as:

$\widehat{S}_0^2 = s_y^2$ (1)

Whichwas unbiased estimator of the population variance:

$S_y^2 = \frac{1}{N-1}\left[\sum_{i=1}^{N}(y_i - \overline{Y})^2\right]$

and its variance was defined as:

$V(\widehat{S}_0^2) = \gamma S_y^4\left(\beta_{2(y)} - 1\right)$ (2)

Firstly, a ratio estimator for population variance using auxiliary information in simple random sampling was proposed (Isaki, 1983):

$\widehat{S}_1^2 = s_y^2 \frac{S_x^2}{s_x^2}$ (3)

Bias and Mean square error of ratio type variance estimatorwas $\widehat{S}_1^2$ :

$\text{Bias}(\widehat{S}_1^2) = \gamma S_y^2\left[\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right]$ (4)

$\text{MSE}(\widehat{S}_1^2) = \gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + \left(\beta_{2(x)} - 1\right) - 2\left(\lambda_{22} - 1\right)\right]$ (5)

An improvement in variance estimation using auxiliary information in simple random sampling was proposed using coefficient of variation(Kadilar and Cingi, 2006):

$\widehat{S}_2^2 = s_y^2\left[\frac{S_x^2 + C_x}{s_x^2 + C_x}\right]$ (6)

$A_2 = \frac{S_x^2}{S_x^2 + C_x}$ where

Bias and Mean square error of ratio type variance estimator was $\widehat{S}_2^2$ :

$\text{Bias}(\widehat{S}_2^2) = \gamma S_y^2 A_1\left[A_1\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right]$ (7)

$\text{MSE}(\widehat{S}_2^2) = \gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + A_1^2\left(\beta_{2(x)} - 1\right) - 2A_1\left(\lambda_{22} - 1\right)\right]$ (8)

An improved exponential estimator for population variance using auxiliary variable in simple random sampling was proposed (Singh *et al.*, 2011). The proposed exponential ratio type variance estimator for population variance was:

$\widehat{S}_3^2 = s_y^2 \exp\left[\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right]$ (9)

Bias and Mean square error of exponential ratio type variance estimator was $\widehat{S}_3^2$ :

$\text{Bias}(\widehat{S}_3^2) = \gamma S_y^2\left[\frac{3}{8}\left(\beta_{2(x)} - 1\right) - \frac{1}{2}\left(\lambda_{22} - 1\right)\right]$ (10)

$\text{MSE}(\widehat{S}_3^2) = \gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + \frac{1}{4}\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right]$ (11)

**Notations and AnExisting Estimator For population Variance in ACS:** **A**ssume a finite population of N units were labelled as 1,2,3,…,N and apreliminary sample of n units wasdrawnbysrswor. Let $w_{yi}$ and $w_{xi}$ the average values in the network which included unit i such that

$w_{yi} = \frac{1}{m_i}\sum_{j \in A_i} y_j$ and $w_{xi} = \frac{1}{m_i}\sum_{j \in A_i} x_j$ respectively. ACSwasmeasured as srswor(Thompson, 1992).

The sample means of the variable of interest and auxiliary variable in the transformed population considered were $\overline{w}_y = \frac{1}{n}\sum_{i=1}^{n} w_{yi}$ and $\overline{w}_x = \frac{1}{n}\sum_{i=1}^{n} w_{xi}$ respectively. Also, consider $S_{wy}^2 = \frac{1}{N-1}\left[\sum_{i=1}^{N}(w_{yi} - \overline{Y})^2\right]$

and $S_{wx}^2 = \frac{1}{N-1}\left[\sum_{i=1}^{N}(w_{xi} - \overline{X})^2\right]$ represents population variance of the study and auxiliary variables in transformed population respectively.Similarly

$$s_{wy}^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}(w_{yi}-\overline{w}_y)^2\right] \quad \text{and} \quad s_{wx}^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}(w_{xi}-\overline{w}_x)^2\right]$$

represents sample variance of the study and auxiliary variables in transformed population respectively. Similarly $C_{wy}$ and $C_{wx}$ represents population coefficient of variations of the study and auxiliary variables in transformed population respectively. The notations to be used are described below:

$$\mu_{wrs} = \frac{1}{N-1}\sum_{i=1}^{N}(w_{yi}-\overline{Y})^r(w_{xi}-\overline{X})^s$$

$$\lambda_{wrs} = \frac{\mu_{wrs}}{\mu_{w20}^{r/2}\mu_{w02}^{s/2}} \quad \text{and} \quad \lambda_{w22} = \frac{\mu_{w22}}{\mu_{w20}\mu_{w02}}$$

$$\beta_{2(wy)} = \frac{\mu_{w40}}{\mu_{w20}^2} \quad \text{and} \quad \beta_{2(wx)} = \frac{\mu_{w04}}{\mu_{w02}^2}$$

Where $\lambda_{w22}$ represent population correlation coefficients between $w_y$ and $w_x$, $\lambda_{w40} = \beta_{2(wy)}$ and $\lambda_{w04} = \beta_{2(wx)}$ are the kurtosis for the population of the study variable and the auxiliary variable respectively, when averages of networks are considered .
Let us define,

$$e_{w0} = \frac{s_{wy}^2 - S_{wy}^2}{S_{wy}^2}, \quad e_{w1} = \frac{s_{wx}^2 - S_{wx}^2}{S_{wx}^2} \quad (12)$$

Where $e_{w0}$ and $e_{w1}$ are the relative sampling errors of the variable of interest and auxiliary variable in transformed population respectively, such that:

$$E(e_{w0}) = E(e_{w1}) = 0, \quad E(e_{w0}e_{w1}) = \gamma(\lambda_{w(22)}-1) \quad (13)$$

$$E(e_{w0}^2) = \gamma(\beta_{2(wy)}-1), \quad E(e_{w1}^2) = \gamma(\beta_{2(wx)}-1) \quad (14)$$

The usual sample variance estimator of the population variance in ACS was defined by (Thompson, 1992) as:

$$\widehat{S}_{w4}^2 = s_{wy}^2 \quad (15)$$

which is unbiased estimator of the population variance in ACS,

$$S_{wy}^2 = \frac{1}{N-1}\left[\sum_{i=1}^{N}(w_{yi}-\overline{Y})^2\right]$$

with its variance:

$$V(\widehat{S}_{w4}^2) = \gamma S_{wy}^4\left(\beta_{2(wy)}-1\right) \quad (16)$$

**Proposed Ratio Estimators for Population Variance in Adaptive Cluster Sampling:** Following (Isaki, 1983) the proposed ratio type variance estimatorin ACS:

$$\widehat{S}_{w5}^2 = s_{wy}^2\frac{S_{wx}^2}{s_{wx}^2} \quad (17)$$

Following (Kadilar andCingi, 2006) another proposed ratio type variance estimator:

$$\widehat{S}_{w6}^2 = s_{wy}^2\left[\frac{S_{wx}^2 + C_{wx}}{s_{wx}^2 + C_{wx}}\right] \quad (18)$$

Following (Singh *et al.*,2011)the proposed exponential ratio type variance estimators:

$$\widehat{S}_{w7}^2 = s_{wy}^2\exp\left[\frac{S_{wx}^2 - s_{wx}^2}{S_{wx}^2 + s_{wx}^2}\right] \quad (19)$$

**Bias and Mean Square Error of Ratio Type Variance Estimator $\widehat{S}_{w5}^2$ :**

In order to drive bias of ratio type variance estimator $\widehat{S}_{w5}^2$ by using the estimator(17) was written as:

$$\widehat{S}_{w5}^2 = \frac{(1+e_{w0})S_{wy}^2 S_{wx}^2}{(1+e_{w1})S_{wx}^2} \quad (20)$$

$$\widehat{S}_{w5}^2 = S_{wy}^2[1+e_{w0}-e_{w1}-e_{w0}e_{w1}+e_{w1}^2] \quad (21)$$

Apply expectation on both side of (6.2) and using the notation (12and13) we obtained,

$$\text{Bias}(\widehat{S}_{w5}^2) = \gamma S_{wy}^2\left[\left(\beta_{2(wx)}-1\right)-\left(\lambda_{w(22)}-1\right)\right] \quad (22)$$

In order to drive mean square error of (17) we have (23) by ignoring the term degree 2 or greater as,

$$\widehat{S}_{w5}^2 = S_{wy}^2[1+e_{w0}-e_{w1}] \quad (23)$$

$$\widehat{S}_{w5}^2 - S_{wy}^2 = S_{wy}^2[e_{w0}-e_{w1}] \quad (24)$$

Taking expectation and squaring on both side of(24) the obtained as,

$$\text{MSE}(\widehat{S}_{w5}^2) = \gamma S_{wy}^4\left[\left(\beta_{2(wy)}-1\right)+\left(\beta_{2(wx)}-1\right)-2\left(\lambda_{w(22)}-1\right)\right] \quad (25)$$

**Bias and Mean Square Error of Ratio Type Variance Estimator $\widehat{S}_{w6}^2$ :**

In order to drive bias of ratio type variance estimator $\widehat{S}_{w6}^2$ by using the estimator (18) may be written as follows:

$$\widehat{S}_{w6}^2 = (1+e_{w0})S_{wy}^2\left[\frac{S_{wx}^2 + C_{wx}}{S_{wx}^2 + C_{wx} + e_{w1}S_{wx}^2}\right] \quad (26)$$

$$\hat{S}^2_{w6} = (1+e_{w0})S^2_{wy}\left[1+\frac{e_{w1}S^2_{wx}}{S^2_{wx}+C_{wx}}\right]^{-1} \quad (27)$$

$$\hat{S}^2_{w6} = (1+e_{w0})S^2_{wy}\left[1+e_{w1}A_2\right]^{-1} \quad (28)$$

where $\quad A_2 = \dfrac{S^2_{wx}}{S^2_{wx}+C_{wx}}$

Opening the terms up-to the second degree, we got (29) as follows:

$$\hat{S}^2_{w6} = (1+e_{w0})S^2_{wy}\left[1-e_{w1}A_2+e^2_{w1}A^2_2\right] \quad (29)$$

Simplifying, discarding the terms three or above we got as follows:

$$\hat{S}^2_{w6} - S^2_{wy} = S^2_{wy}\left[e_{w0}-e_{w1}A_2-e_{w0}e_{w1}A_2+e^2_{w1}A^2_2\right] \quad (30)$$

Apply expectation on both sides of(30) and using the notations(12and 13) we got as follows:

$$\text{Bias}(\hat{S}^2_{w6}) = \gamma S^2_{wy}A_2\left[A_2\left(\beta_{2(wx)}-1\right)-\left(\lambda_{w(22)}-1\right)\right] \quad (31)$$

To drive mean squared error of(18) we have(32) by discarding the term degree two or above asfollows:

$$\hat{S}^2_{w6} = S^2_{wy}\left[1+e_{w0}-e_{w1}A_2\right] \quad (32)$$

$$\hat{S}^2_{w6} - S^2_{wy} = S^2_{wy}\left[e_{w0}-e_{w1}A_2\right] \quad (33)$$

Apply expectation and squaring on both sides (33) we obtained:

$$\text{MSE}(\hat{S}^2_{w6}) = \gamma S^4_{wy}\left[\left(\beta_{2(wy)}-1\right)+A^2_2\left(\beta_{2(wx)}-1\right)-2A_2\left(\lambda_{w(22)}-1\right)\right] \quad (34)$$

**Bias and Mean Square Error of Exponential Ratio Type Variance Estimator $\hat{S}^2_{w7}$ :**

In order to drive bias of exponential ratio type variance estimator $\hat{S}^2_{w7}$ the estimator(19) may be written as follows:

$$\hat{S}^2_{w7} = S^2_{wy}(1+e_{w0})\exp\left[\frac{S^2_{wx}-(1+e_{w1})S^2_{wx}}{S^2_{wx}+(1+e_{w1})S^2_{wx}}\right] \quad (35)$$

$$\hat{S}^2_{w7} = S^2_{wy}(1+e_{w0})\exp\left[-\frac{e_{w1}}{2}+\frac{e^2_{w1}}{4}\right] \quad (36)$$

Opening the exponential term up-to second degree, we got(37) as follows:

$$\hat{S}^2_{w7} = S^2_{wy}(1+e_{w0})\left[1-\frac{e_{w1}}{2}+\frac{e^2_{w1}}{4}+\frac{e^2_{w1}}{8}\right] \quad (37)$$

Simplification, discarding the terms with degree three or more we get as follows:

$$\hat{S}^2_{w7} - S^2_{wy} = S^2_{wy}\left[-\frac{e_{w1}}{2}+\frac{e^2_{w1}}{4}+\frac{e^2_{w1}}{8}+e_{w0}-\frac{e_{w0}e_{w1}}{2}\right] \quad (38)$$

Applying expectation on both sides of(38) and using notations(12and13) as follows:

$$\text{Bias}(\hat{S}^2_{w7}) = \gamma S^2_{wy}\left[\frac{3}{8}\left(\beta_{2(wx)}-1\right)-\frac{1}{2}\left(\lambda_{w(22)}-1\right)\right] \quad (39)$$

To drive mean squared error of(19) we have(40) as follows:

$$\hat{S}^2_{w7} = S^2_{wy}(1+e_{w0})\exp\left[\frac{-e_{w1}}{2}\right] \quad (40)$$

Expanding the exponential term, discarding terms with power two or above, we got(41) as follows:

$$\hat{S}^2_{w7} = S^2_{wy}(1+e_{w0})\left[1-\frac{e_{w1}}{2}\right] \quad (41)$$

$$\hat{S}^2_{w7} - S^2_{wy} = S^2_{wy}(1+e_{w0})\left[e_{w0}-\frac{e_{w1}}{2}\right] \quad (42)$$

Applying square and expectations on both sides of(42) and using notation (12and 13) we got asfollows:

$$\text{MSE}(\hat{S}^2_{w7}) = \gamma S^4_{wy}\left[\left(\beta_{2(wy)}-1\right)+\frac{1}{4}\left(\beta_{2(wx)}-1\right)-\left(\lambda_{w(22)}-1\right)\right] \quad (43)$$

**SimulationsPerformance:** Anactual population was considered and simulations executedto measure the efficiency of proposed estimators with the other estimators. Ten thousand iterations was performed for all estimators to got accuracy estimates with srswor and the preliminary sample size of 5,10,15,20 and 25, for the simulations study.In ACS,the ultimate sample size wasgenerallybigger than the preliminary sample size. The estimatedultimate sample size in ACS wasdenoted by E(v),was sum of the inclusion probabilities of all the quadrats,

$$E(v) = \sum_{i=1}^{N}\pi_i \quad (44)$$

In ACS the expected final sample size varies from sample to sample. For the comparison, the sample variance from srswor based on E(v) has variance using the formula:

$$Var(\hat{s}^2_y) = \frac{S^4_y(N-E(v))}{NE(v)}(\beta_{2(x)}-1) \quad (45)$$

The estimated relative bias of the estimated variance is defined as:

$$RBias\left(\hat{S}^2_*\right) = \frac{\frac{1}{r}\sum_{i=1}^{r}(\hat{S}^2_*)-S^2_*}{S^2_*} \quad (46)$$

Where $\hat{S}_*^2$ was the magnitude of the relevant estimator for the sample i and the number of iterations was represented by r.

The estimated mean squared error of the estimated variance:

$$MSE(\hat{S}_*^2) = \frac{1}{r}\sum_{i=1}^{r}(\hat{S}_*^2 - S_*^2)$$  (47)

Thepercentage relative efficiency is defined as:

$$PRE = \frac{Var(\hat{S}_y^2)}{MSE(\hat{S}_*^2)} * 100$$  (48)

**Population:**Real population comprisedof blue-winged teal (BWT) data and green-winged teal (GWT) data collected (Smith *et al.,*1995)were counts of two species of waterfowl in 50 100-$km^2$ quadrats in central Florida. The green-winged teal (Table 1) data was taken as the study variable and the condition was imposed on the auxiliary variable BWT (Table 2) as $C_x > 10$ to added unit in the sample. The x-values wereattained and averaged (Table 3) for keeping the sample network in accordancewith the pre definedcondition and for every sample network y-values were attained and averaged(Table 4).

**Table 1.Green-winged teal data as study variable.**

| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 35 | 75 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2255 | 13 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 24 |

**Table 2.Blue-winged teal data as auxiliary variable.**

| 0 | 0 | 3 | 5 | 0 | 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 24 | 14 | 0 | 0 | 10 | 103 | 0 |
| 0 | 0 | 0 | 0 | 2 | 3 | 2 | 0 | 13639 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 14 | 122 |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 177 |

**Table 3.Transformed blue-winged teal data as auxiliary variable.**

| 0 | 0 | 3 | 5 | 0 | 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 19 | 19 | 0 | 0 | 10 | 2811 | 0 |
| 0 | 0 | 0 | 0 | 2 | 3 | 2 | 0 | 2811 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2811 | 2811 |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 2811 |

**Table 4.Transformed green-winged teal data as study variable.**

| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 35 | 470.80 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 470.80 | 13 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 470.80 | 470.80 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 470.80 |

## RESULTS AND DISCUSSIONS

High positive correlation (0.99)between both types of birdswas found and correlation remainedunchanged in the transformed population. So, there found a high correlation between the sampling unit level and network level. Theconventionalestimators in srsworexecutedwell than ACS estimators for high correlation at sampling unit level and lower at high correlationat network level (Dryver and Chao, 2007).

The overall variance of the variable of interest was 101576.10 and for auxiliary 3716168 while in the transformed population variances reduced to 20291.12 and 724927.10 respectively.The within network variance of the variable of interest for the network (75, 2255, 0, 0, 24) was 995740.70with corresponding values of the auxiliary variable within network variance 36642684. Therefore, adaptive estimators were anticipated to executesuperior and more efficient than usual comparable estimators. The adaptive estimators were more efficient

than the usual estimators if within-network variances account a large portion of the overall variance (Dryver and Chao, 2007).

**Table 5.Estimated Relative Bias.**

| n | $\hat{S}_0^2$ | $\hat{S}_1^2$ | $\hat{S}_2^2$ | $\hat{S}_3^2$ | $\hat{S}_4^2$ | $\hat{S}_5^2$ | $\hat{S}_6^2$ | $\hat{S}_7^2$ |
|---|---|---|---|---|---|---|---|---|
| 5 | -0.03 | * | 1.44 | -0.55 | 0.00 | * | 1.23 | -0.33 |
| 10 | 0.02 | * | 1.60 | -0.47 | 0.00 | * | 0.68 | -0.17 |
| 15 | 0.02 | * | 1.49 | -0.41 | 0.00 | * | 0.37 | -0.09 |
| 20 | 0.02 | 12.50 | 1.33 | -0.35 | -0.01 | 7.76 | 0.14 | -0.06 |
| 25 | 0.00 | 6.86 | 1.09 | -0.28 | 0.00 | 2.49 | 0.06 | -0.03 |

The results of estimated percentage relative efficiencies in the comparable sample sizes in (Table 6) showed the inferior performance of usual estimators under srswordueto the larger variances of the networks of the variable of study. The usual sample variance estimator in ACS performed better as compare with the proposed estimators $\hat{S}_5^2$ and $\hat{S}_6^2$ .The comparative percentage relative efficiencies increase as the sample size increases for the proposed adaptive estimators $\hat{S}_6^2$ and $\hat{S}_7^2$ .

**Table 6.Estimated Percentage Relative Efficiency.**

| E(v) | $\hat{S}_0^2$ | $\hat{S}_1^2$ | $\hat{S}_2^2$ | $\hat{S}_3^2$ | $\hat{S}_4^2$ | $\hat{S}_5^2$ | $\hat{S}_6^2$ | $\hat{S}_7^2$ |
|---|---|---|---|---|---|---|---|---|
| 10.57 | 40.20 | * | 13.65 | 168.90 | 6091.0 | * | 217.31 | 12130.25 |
| 18.46 | 39.45 | * | 8.98 | 124.00 | 6495.1 | * | 375.72 | 11024.60 |
| 24.54 | 41.29 | * | 7.89 | 101.15 | 6868.7 | * | 671.54 | 12552.49 |
| 29.45 | 43.39 | 0.02 | 7.42 | 86.87 | 7055.4 | 0.60 | 1634.47 | 14848.68 |
| 33.60 | 45.89 | 0.04 | 7.83 | 77.44 | 7535.5 | 1.66 | 3914.24 | 18850.34 |

The percentage relative efficiency of proposed exponential ratio estimator $\hat{S}_7^2$ remainedmaximum for all sample sizes in comparison with all the estimators. Thus use of modified exponential ratio estimator $\hat{S}_7^2$ was better in ACS, and exponential type estimators were much suitable and robust for patchy, rare and clustered population. The product and regression estimators for population variance can be studied, moreover the logarithmic type estimators can also be studied as a future research in ACS.

**Table 7. Descriptive Measure of the Population**

| | | | |
|---|---|---|---|
| $\bar{X} = 282.42$ | $S_x^2 = 3716168$ | $C_x = 6.83$ | $\rho_{xy} = 0.99$ |
| $\bar{Y} = 48.04$ | $S_y^2 = 101576.10$ | $C_y = 6.64$ | $\rho_{wxwy} = 0.99$ |
| $\bar{w}_x = 282.42$ | $S_{wx}^2 = 724927.10$ | $C_{wx} = 3.02$ | $N = 50$ |
| $\bar{w}_y = 48.04$ | $S_{wy}^2 = 20291.12$ | $C_{wy} = 2.97$ | $C_x > 10$ |

**Conclusions:**In this simulation study 0/0 was not treated as 0. The classical ratio estimator $\hat{S}_1^2$ and proposed ratio estimator $\hat{S}_5^2$ in ACS did not perform and return no value (*) for the initial sample sizes 5, 10 and 15. As the sample size increases the estimated relative bias (Table 5) of all the estimators' decreases. The usual sample variance estimator $\hat{S}_4^2$ showed that amount of estimated relative bias is zero so it is an unbiased estimator for population variance in ACS. The proposed ratio estimator $\hat{S}_5^2$ produced no value (*) for the sample sizes 5, 10 and 15 just like ratio estimator $\hat{S}_1^2$ . At sample size 20 both ratio estimators $\hat{S}_1^2$ and $\hat{S}_5^2$ showed amount of biased but this amount decreased sharply at sample size 25. The proposed estimators $\hat{S}_6^2$ also showed the large value at the initial sample size 5 but this amount sharply declined for larger sample sizes. The proposed estimators $\hat{S}_7^2$ showed negative amount but this also reduces for larger sample sizes. Thus it was suggested to include a large sample size for the small bias in ACS, as it happened in simple random sampling.

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