INTUITIONISTIC FUZZY BCI-COMMUTATIVE IDEALS IN BCI-ALGEBRAS

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ABSTRACT: In this paper, we consider the intuitionistic fuzzification of the concept of BCI-commutative ideals in BCI-algebras and investigate some of their properties. We will show that every intuitionistic fuzzy BCI-commutative ideal of a BCI-algebra X is an intuitionistic fuzzy ideal of X. We will give some characterization theorems on this intuitionistic fuzzy ideal using the concept of upper t-level cut and lower s-level cut. We will also give its characterization using the concept of intuitionistic fuzzy ideal extensions.

2000 Mathematics subject classification: 03G25, 06F35.

Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy ideal, Intuitionistic fuzzy BCI-commutative ideal, Intuitionistic fuzzy ideal extension.

INTRODUCTION AND PRELIMINARIES

After the introduction of the concept of fuzzy sets by Zadeh (Zadeh, 1965), several researches were conducted on the generalization of the notion of fuzzy sets. The idea of "intuitionistic fuzzy set" was first published by Atanassov, as a generalization of the notion of fuzzy sets (Atanassov, 1986). After that many researchers considered the fuzzification of ideals and subalgebras in BCK (BCI)-algebras. Jun and Meng introduced the concept of fuzzy BCI-commutative ideals in BCI-algebras (Jun and Meng, 1994). Zhan and Tan introduced the concept of intuitionistic fuzzy α-ideals in BCI-algebras (Zhan and Tan, 2004). Satyanarayana, Madhavi and Prasad introduced the concept of intuitionistic fuzzy H-ideals in BCK-algebras (Satyanarayana et al., 2010). As a continuation of the above two, we further discuss the intuitionistic fuzzification of the concept of BCI-commutative ideals in BCI-algebras and investigate some of their properties.

Let us recall that an algebra $(X,*,0)$ of type $(2,0)$ is called a BCI-algebra if it satisfies the following conditions:

$((x*y)*(x*z))*(z*y)=0$

$(x*(x*y))*y=0$

$x*x=0$

$x*y=0$ and $y*x=0 \Rightarrow x=y$

for all $x,y,z\in X$.

In a BCI-algebra, we can define a partial ordering "$\leq$" by $x\leq y$ if and only if $x*y=0$. In a BCI-algebra X, the set $M=\{x\in X \mid 0*x=0\}$ is a sub algebra and is called the BCK-part of X. A BCI-algebra X is called proper if $X-M\neq\Phi$. Otherwise it is improper. Moreover, in a BCI-algebra the following conditions hold:

$(x*y)*z=(x*z)*y$

$x*0=x$

$x\leq y \Rightarrow x*z\leq y*z$ and $z*y\leq z*x$

$0*(x*y)=(0*x)*(0*y)$

$0*(0*(x*y))=0*(y*x)$

$(x*z)*(y*z)\leq x*y$

Now here after giving some known results, we will define intuitionistic fuzzy BCI-commutative ideals in BCI-algebras and investigate some of their properties.

Lemma 1.1(Jun and Kim, 2000) Let IFSA= (α_A, β_A) be an intuitionistic fuzzy ideal of X. If the inequality $x*y\leq z$ holds in X, then $\alpha_A(x)\geq$ min $\{\alpha_A(y),\alpha_A(z)\}$ and $\beta_A(x)\leq$ max $\{\beta_A(y),\beta_A(z)\}$.

Lemma 1.2(Jun and Kim, 2000) Let IFSA= (α_A, β_A) be an intuitionistic fuzzy ideal of X. If the inequality $x\leq y$ holds in X, then $\alpha_A(x)\geq\alpha_A(y)$ and $\beta_A(x)\leq\beta_A(y)$, that is α_A is order reversing while β_A is order preserving.

2. Intuitionistic fuzzy BCI-commutative ideal

An IFSA $=(\alpha_A,\beta_A)$ in a BCI-algebra X is called an intuitionistic fuzzy BCI-commutative ideal of X if it satisfies:

(IFBCI-C-1) $\alpha_A(0)\geq\alpha_A(x)$ and $\beta_A(0)\leq\beta_A(x)$

(IFBCI-C-2)
$\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) \geq$ min $\{\alpha_A((x*y)*z), \alpha_A(z)\}$
(IFBCI-C-3)
$\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) \leq$ max $\{\beta_A((x*y)*z), \beta_A(z)\}$
for all $x, y \in X$.

Example 2.1 Consider the BCI-algebra X = $\{0,1,2,3\}$ with the following cayley table:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	1	0	2
3	3	3	3	0

Define an IFSA $=(\alpha_A, \beta_A)$ in X as follows:
$\alpha_A(0) = \alpha_A(3) = 1$, $\alpha_A(1) = \alpha_A(2) = t$ and
$\beta_A(0) = \beta_A(3) = 0$, $\beta_A(1) = \beta_A(2) = s$
where $s, t \in (0,1)$ and $s + t \leq 1$

By routine calculations it is easy to verify that IFSA $=(\alpha_A, \beta_A)$ is an intuitionistic fuzzy BCI-commutative ideal of X.

Theorem 2.2 Every intuitionistic fuzzy BCI-commutative ideal of a BCI-algebra X is an intuitionistic fuzzy ideal of X.

Proof Assume that IFSA $=(\alpha_A, \beta_A)$ is an intuitionistic fuzzy BCI-commutative ideal of X. Then by definition we have:
(IFBCI-C-2)
$\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) \geq$ min $\{\alpha_A((x*y)*z), \alpha_A(z)\}$
(IFBCI-C-3)
$\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) \leq$ max $\{\beta_A((x*y)*z), \beta_A(z)\}$
for all $x, y \in X$.
Putting z=y and y=0 we get
$\alpha_A(x*((0*(0*x))*(0*(0*(x*0))))) \geq$ min $\{\alpha_A((x*0)*y), \alpha_A(y)\}$ and
$\beta_A(x*((0*(0*x))*(0*(0*(x*0))))) \leq$ max $\{\beta_A((x*0)*y), \beta_A(y)\}$
$\Rightarrow \alpha_A(x) \geq$ min $\{\alpha_A(x*y), \alpha_A(y)\}$ and $\beta_A(x) \leq$ max $\{\beta_A(x*y), \beta_A(y)\}$

Hence IFSA $=(\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of X. Whereas the converse of this theorem may not be true. For this we consider the following example.

Consider the BCI-algebra $X = \{0,1,2,3,4\}$ with the following Cayley table:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	3	4	1	0

Define an IFSA $=(\alpha_A, \beta_A)$ in X by
$\alpha_A(0) = \alpha_A(2) = 1$, $\alpha_A(1) = \alpha_A(3) = \alpha_A(4) = t$ and
$\beta_A(0) = \beta_A(2) = 0$, $\beta_A(1) = \beta_A(3) = \beta_A(4) = s$
where $s, t \in (0,1)$ and $s + t \leq 1$.

By routine calculations it is easy to verify that IFSA $=(\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of X but it is not an intuitionistic fuzzy BCI-commutative ideal of X because:
$\alpha_A(1*((3*(3*1))*(0*(0*(1*3))))) = \alpha_A(1) = t < 1 =$ min $\{\alpha((1*3)*0), \alpha_A(0)\}$.

Theorem 2.3 Let IFSA $=(\alpha_A, \beta_A)$ be an intuitionistic fuzzy ideal of a BCI-algebra X. Then the following conditions are equivalent:

1. IFSA $=(\alpha_A, \beta_A)$ is an intuitionistic fuzzy BCI-commutative ideal of X.

2. $\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) \geq \alpha_A(x*y)$ and $\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) \leq \beta_A(x*y)$

3. $\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) = \alpha_A(x*y)$ and $\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) = \beta_A(x*y)$

Proof ($1 \Rightarrow 2$) Let IFSA $=(\alpha_A, \beta_A)$ be an intuitionistic fuzzy BCI-commutative ideal of X. Then by definition we have:
$\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) \geq$ min $\{\alpha_A((x*y)*z), \alpha_A(z)\}$ and

$\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) \leq_{\max} \{\beta_A((x*y)*z), \beta_A(z)\}$
for all $x, y \in X$.
By putting $z = 0$ we get
$\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) \geq_{\min} \{\alpha_A((x*y)*0), \alpha_A(0)\}$ and
$\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) \leq_{\max} \{\beta_A((x*y)*0), \beta_A(0)\}$ that is
$\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) \geq \alpha_A(x*y)$
and
$\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) \leq \beta_A(x*y)$
which are the required conditions.

($2 \Rightarrow 3$) Assume that
$\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) \geq \alpha_A(x*y)$
(a)
and
$\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) \leq \beta_A(x*y)$
(b)
Since
$(y*(y*x))*(0*(0*(x*y))) = (y*(y*x))*(0*(y*x)) \leq y$
$\Rightarrow x*y \leq x*((y*(y*x))*(0*(0*(x*y))))$
Then by lemma(1.2) we get
$\alpha_A(x*y) \geq \alpha_A(x*((y*(y*x))*(0*(0*(x*y)))))$
(c)
and
$\beta_A(x*y) \leq \beta_A(x*((y*(y*x))*(0*(0*(x*y)))))$
(d)
From (a) and (c) and (b) and (d) we get
$\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) = \alpha_A(x*y)$
and
$\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) = \beta_A(x*y)$
which are the required conditions.

($3 \Rightarrow 1$) Assume that
$\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) = \alpha_A(x*y)$
(a)
and
$\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) = \beta_A(x*y)$
(b)
for all $x, y \in X$.
Since $(x*y)((x*y)*z) \leq z$
Therefore by using lemma (1.1) we get
$\alpha_A(x*y) \geq_{\min} \{\alpha_A((x*y)*z), \alpha_A(z)\}$ (c)
and
$\beta_A(x*y) \leq_{\max} \{\beta_A((x*y)*z), \beta_A(z)\}$ (d)
From (a) and (c) and (b) and (d) we get
$\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) \geq_{\min} \{\alpha_A((x*y)*z), \alpha_A(z)\}$ and
$\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) \leq_{\max} \{\beta_A((x*y)*z), \beta_A(z)\}$.

Hence IFSA $=(\alpha_A, \beta_A)$ is an intuitionistic fuzzy BCI-commutative ideal of X.

For any $t, s \in [0,1]$ and fuzzy sets α_A and β_A in a non-empty set X, the set $U(\alpha_A;t) = \{x \in X \mid \alpha_A(x) \geq t\}$ is called an upper t-level cut of α_A and the set $L(\beta_A;s) = \{x \in X \mid \beta_A(x) \leq s\}$ is called a lower s-level cut of β_A.

Theorem 2.4 An IFSA $=(\alpha_A, \beta_A)$ is an intuitionistic fuzzy BCI-commutative ideal of a BCI-algebra X if and only if the non-empty upper t-level cut $U(\alpha_A;t)$ and the non-empty lower s-level cut $L(\beta_A;s)$ are BCI-commutative ideals of X for any $s, t \in [0,1]$.

Proof Suppose that An IFSA $=(\alpha_A, \beta_A)$ is an intuitionistic fuzzy BCI-commutative ideal of a BCI-algebra X. Since $U(\alpha_A;t) \neq \varnothing$, $L(\beta_A;s) \neq \varnothing$. So for any $x \in U(\alpha_A;t)$ we have
$\alpha_A(x) \geq t \Rightarrow \alpha_A(0) \geq \alpha_A(x) \geq t \Rightarrow 0 \in U(\alpha_A;t)$.
Now let $(x*y)*z \in U(\alpha_A;t)$ and $z \in U(\alpha_A;t)$.
Then $\alpha_A((x*y)*z) \geq t$ and $\alpha_A(z) \geq t$. Since
$\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) \geq_{\min} \{\alpha_A((x*y)*z), \alpha_A(z)\} \geq t \Rightarrow x*((y*(y*x))*(0*(0*(x*y)))) \in U(\alpha_A;t) \Rightarrow U(\alpha_A;t)$
is BCI-commutative ideal of X. Similarly we can prove that $L(\beta_A;s)$ is a BCI-commutative ideal of X.
Conversely suppose that the non-empty upper t-level cut $U(\alpha_A;t)$ and the non-empty lower s-level cut $L(\beta_A;s)$ are BCI-commutative ideals of X for any $s, t \in [0,1]$. If possible assume that there exists some

$x_0 \in X$ such that $\alpha_A(0) < \alpha_A(x_0)$ and $\beta_A(0) > \beta_A(x_0)$. Put $t_0 = 1/2\{\alpha_A(0) + \alpha_A(x_0)\}$ then $\alpha_A(0) < t_0 < \alpha_A(x_0) \Rightarrow x_0 \in U(\alpha_A;t_0)$ and 0 does not belong to $U(\alpha_A;t_0)$ which is a contradiction to the fact that $U(\alpha_A;t_0)$ is a BCI-commutative ideal of X. Therefore we must have $\alpha_A(0) \geq \alpha_A(x)$ for all $x \in X$. Similarly by putting $s_0 = 1/2\{\beta_A(0) + \beta_A(x_0)\}$ we can prove that $\beta_A(0) \leq \beta_A(x)$ for all $x \in X$. If possible assume that there exists some $x_0, y_0, z_0 \in X$ such that p= $\alpha_A(x_0 * ((y_0 * (y_0 * x_0)) * (0 * (0 * (x_0 * y_0))))) <$ min $\{\alpha_A((x_0 * y_0) * z_0), \alpha_A(z_0)\} = q$. Put $t_0 = 1/2\{p + q\}$ then $p < t_0 < q \Rightarrow (x_0 * y_0) * z_0 \in U(\alpha_A;t_0)$ and $z_0 \in U(\alpha_A;t_0)$ whereas $x_0 * ((y_0 * (y_0 * x_0)) * (0 * (0 * (x_0 * y_0))))$ does not belong to $U(\alpha_A;t_0)$ which is a contradiction to the fact that $U(\alpha_A;t_0)$ is a BCI-commutative ideal of X. Therefore $\alpha_A(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \geq$ min $\{\alpha_A((x * y) * z), \alpha_A(z)\}$ for all $x, y, z \in X$. Similarly we can prove that $\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) \leq max\{\beta_A((x*y)*z), \beta_A(z)\}$ for all $x, y, z \in X$. Hence IFSA $= (\alpha_A, \beta_A)$ is an intuitionistic fuzzy BCI-commutative ideal of X.

Theorem 2.5 Let IFSA $= (\alpha_A, \beta_A)$ be an intuitionistic fuzzy closed ideal of a BCI-algebra X. Then IFSA $= (\alpha_A, \beta_A)$ is an intuitionistic fuzzy BCI-commutative ideal of X if and only if

(a) $\alpha_A(x * (y * (y * x))) \geq \alpha_A(x * y)$

(b) $\beta_A(x * (y * (y * x))) \leq \beta_A(x * y)$

for all $x, y, z \in X$.

Proof Suppose that IFSA $= (\alpha_A, \beta_A)$ is an intuitionistic fuzzy BCI-commutative ideal of X. Since IFSA $= (\alpha_A, \beta_A)$ is an intuitionistic fuzzy closed ideal of X, so we have $\alpha_A(0 * (x * y)) \geq \alpha_A(x * y)$ and $\beta_A(0 * (x * y)) \leq \beta_A(x * y)$. Now

$(x*(y*(y*x)))*(x*((y*(y*x))*(0*(0*(x*y))))) \leq ((y*(y*x))*(0*(0*(x*y))))*(y*(y*x)) = ((y*(y*x))*(y*(y*x)))*(0*(0*(x*y))) = 0*(0*(0*(x*y))) = (0*(x*y))$

Hence by lemma(1.1) we have

$\alpha_A(x * (y * (y * x))) \geq$ min $\{\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))), \alpha_A(0*(x*y))\}$

and $\beta_A(x * (y * (y * x))) \leq$ max $\{\beta_A(x*((y*(y*x))*(0*(0*(x*y))))), \beta_A(0*(x*y))\}$

Now by using theorem(2.3(2)) we have

$\alpha_A(x * (y * (y * x))) \geq$ min $\{\alpha_A(x * y), \alpha_A(0 * (x * y))\}$ = $\alpha_A(x * y)\}$ and

$\beta_A(x * (y * (y * x))) \leq$ max $\{\beta_A(x * y), \beta_A(0 * (x * y))\}$ = $\beta_A(x * y$. Which are the required conditions.

Now conversely suppose that IFSA $= (\alpha_A, \beta_A)$ be is intuitionistic fuzzy closed ideal of X satisfying the conditions:

$\alpha_A(x * (y * (y * x))) \geq \alpha_A(x * y)$

$\beta_A(x * (y * (y * x))) \leq \beta_A(x * y)$

for all $x, y, z \in X$.

Consider

$(x*((y*(y*x))*(0*(0*(x*y)))))*(x*(y*(y*x))) \leq (y*(y*x))*((y*(y*x))*(0*(0*(x*y)))) \leq 0*(0*(x*y))$ By using lemma (1.1) we get

$\alpha_A(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \geq$ min $\{\alpha_A(x * (y * (y * x))), \alpha_A(0 * (0 * (x * y))))\geq$ min $\{\alpha_A(x * y), \alpha_A((0 * (x * y)))) = \alpha_A(x * y)$ (By given conditions)

and

$\beta_A(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \leq$ max $\{\beta_A(x * (y * (y * x))), \beta_A(0 * (0 * (x * y)))) \leq$ max $\{\beta_A(x * y), \beta_A((0 * (x * y)))) = \beta_A(x * y)$ (By given conditions)

Hence by theorem(2.3) IFSA $= (\alpha_A, \beta_A)$ is an intuitionistic fuzzy BCI-commutative ideal of X.

2.6. Definition (Satyanarayana et al., 2010) Let f be a mapping on a set X and A $= (\alpha_A, \beta_A)$ be an intuitionistic fuzzy set in X. Then the fuzzy sets u and v on f(X) defined by $u(y) = Sup_{x \in f^{-1}(y)} \alpha_A(x)$ and

$v(y) = Inf_{x \in f^{-1}(y)} \beta_A(x)$, for all $y \in f(X)$, are called the images of A under f. If u, v are fuzzy sets in $f(X)$ then the fuzzy sets $\alpha_A = uof$ and $\beta_A = vof$ are called the pre-images of u and v under f.

Theorem 2.7 Let $f : X \mapsto X'$ be an onto homomorphism of BCI-algebras. If $A' = (u,v)$ is an intuitionistic fuzzy BCI-commutative ideal of BCI-algebra X' then the pre-image of $A' = (u,v)$ under f is an intuitionistic fuzzy BCI-commutative ideal of X.

Proof Let an IFSA $= (\alpha_A, \beta_A)$ where $\alpha_A = uof$ and $\beta_A = vof$ be the pre-image of $A' = (u,v)$ under f. Since $A' = (u,v)$ is an intuitionistic fuzzy BCI-commutative ideal of X', we have $u(0') \geq u(f(x)) = \alpha_A(x)$ and $v(0') \leq v(f(x)) = \beta_A(x)$. On the other hand $u(0') = u(f(0)) = \alpha_A(0)$ and $v(0') = v(f(0)) = \beta_A(0)$. Therefore $\alpha_A(0) \geq \alpha_A(x)$ and $\beta_A(0) \leq \beta_A(x)$, for all $x \in X$. Now we show that $\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) \geq$ min $\{\alpha_A((x*y)*z), \alpha_A(z)\}$ and $\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) \leq$ max $\{\beta_A((x*y)*z), \beta_A(z)\}$ for all $x, y, z \in X$.

Now $\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) = u(f(x*((y*(y*x))*(0*(0*(x*y)))))) = u(f(x)*((f(y)*(f(y)*f(x)))*(f(0)*(f(0)*(f(x)*f(y)))))) \geq$ min $\{u((f(x)*f(y))*z'), u(z')\}$.

Since f is an onto homomorphism, so there exists some $z \in X$ such that $f(z) = z'$. Thus $\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) \geq$ min $\{u((f(x)*f(y))*f(z)), u(f(z))\}$ = min $\{u(f((x*y)*z)), u(f(z))\}$ = min $\{\alpha_A((x*y)*z), \alpha_A(z)\}$. Therefore the result $\alpha_A(x*((y*(y*x))*(0*(0*(x*y))))) \geq$ min $\{\alpha_A((x*y)*z), \alpha_A(z)\}$, is true for all $x, y, z \in X$ because z' is an arbitrary element of X' and f is an onto mapping. Similarly we can prove that $\beta_A(x*((y*(y*x))*(0*(0*(x*y))))) \leq$ max $\{\beta_A((x*y)*z), \beta_A(z)\}$ for all $x, y, z \in X$. Hence the pre-image A $= (\alpha_A, \beta_A)$ of $A' = (u,v)$ under f is an intuitionistic fuzzy BCI-commutative ideal of X.

3. Fuzzy ideal extensions (Jeong, 2002) Let μ be a fuzzy subset of a BCK-algebra X and $a \in X$. Then the fuzzy subset $< \mu, a >: X \mapsto [0,1]$ defined by $< \mu, a > (x)$ = $\mu(x*a)$ is called the extension of μ by a.

3.1 Intuitionistic Fuzzy ideal extensions(Jeong, 2010) Let (α_A, β_A) be an intuitionistic fuzzy set in a BCK-algebra X and $a, b \in X$. Then the intuitionistic fuzzy set $< (\alpha_A, \beta_A), (a,b) >$ defined by: $< (\alpha_A, \beta_A), (a,b) > = (< \alpha_A, a >, < \beta_A, b >)$ is called the extension of (α_A, β_A) by (a,b). If $a = b$, then it is denoted by $< (\alpha_A, \beta_A), a >$.

Theorem 3.2 Let an IFSA $= (\alpha_A, \beta_A)$ be intuitionistic fuzzy BCI-commutative ideal of a positive implicative BCK-algebra X and $a, b \in X$. Then the extension $< (\alpha_A, \beta_A), (a,b) >$ of (α_A, β_A) by (a,b) is also an intuitionistic fuzzy BCI-commutative ideal of X.

Proof Suppose that an IFSA $= (\alpha_A, \beta_A)$ be intuitionistic fuzzy BCI-commutative ideal of a positive implicative BCK-algebra X and $a, b \in X$. Let $x, y \in X$. Then we have $< \alpha_A, a > (0) = \alpha_A(0*a) = \alpha_A(0) \geq \alpha_A(x*a) = < \alpha_A, a > (x)$ and $< \beta_A, b > (0) = \beta_A(0*b) = \beta_A(0) \leq \beta_A(x*b) = < \beta_A, b > (x)$.

Now $< \alpha_A, a > (x*((y*(y*x))*(0*(0*(x*y))))) = \alpha_A((x*((y*(y*x))*(0*(0*(x*y)))))*a) = \alpha_A(((x*a)*(((y*a)*((y*a)*(x*a)))*((0*a)*((0*a)*((x*a)*(y*a)))))) = \alpha_A(((x*a)*(((y*a)*((y*a)*(x*a)))*(0*(0*((x*a)*(y*a)))))) \geq$ min $\{\alpha_A(((x*a)*(y*a))*(z*a)), \alpha_A(z*a)\}$ = min $\{\alpha_A(((x*y)*z)*a), \alpha_A(z*a)\}$ = min $\{< \alpha_A, a > (((x*y)*z)), < \alpha_A, a > (z)\}$.

Similarly $<\beta_A$, $b>(x*((y*(y*x))*(0*(0*(x*y)))))$ = $\beta_A((x*((y*(y*x))*(0*(0*(x*y)))))*b)$ = $\beta_A(((x*b)*(((y*b)*((y*b)*(x*b)))*((0*b)*((0*b)*((x*b)*(y*b))))))$ = $\beta_A(((x*b)*(((y*b)*((y*b)*(x*b)))*(0*(0*((x*b)*(y*b))))))\leq$ max$\{\beta_A(((x*b)*(y*b))*(z*b))$, $\beta_A(z*b)\}$ = max$\{\beta_A(((x*y)*z)*b)$, $\beta_A(z*b)\}$ = max $\{<\beta_A,b>(((x*y)*z)),<\beta_A,b>(z)\}$.

Hence the extension $<(\alpha_A,\beta_A)$, $(a,b)>$ of (α_A,β_A) by (a,b) is an intuitionistic fuzzy BCI-commutative ideal of X.

RESULTS AND DISCUSSION

It is clear that ideals with special properties play an important role in the study of the structure of an algebraic system. In this paper, we studied the notions of intuitionistic fuzzy BCI-commutative ideals in BCI-algebras and investigated some of their properties. We proved that every intuitionistic fuzzy BCI-commutative ideal of a BCI-algebra X is an intuitionistic fuzzy ideal of X. We characterized intuitionistic fuzzy BCI-commutative ideals using the concept of upper t-level cut and lower s-level cut and also using the concept of intuitionistic fuzzy ideal extensions. The results can be applied to other algebraic structures. It is our hope that this work will serve as a foundation for further study of the theory of BCK/BCI-algebras.

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