CALCULATION OF IDEAL FLOW AROUND A JOUKOWSKI AEROFOIL USING INDIRECT PANEL METHOD WITH DOUBLET DISTRIBUTION ALONE

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ABSTRACT: In this research paper, indirect panel method is applied to calculate an ideal flow around two dimensional body i.e. Joukowski aerofoil with constant approach using doublet distribution alone for which the analytical solution is available. To check the accuracy of the method, the computed velocity is compared to the analytical result for the ideal flow around a joukowski aerofoil.

Key words: Indirect Panel Method, Joukowski Aerofoil, Ideal Flow.

INTRODUCTION

The significance of the panel method has been generally familiar during the last few decades. Now a day, the panel method is magnificently applied in numerical community. The panel method consists of sub-dividing the boundary of the body into a series of discrete panel, over which the function can fluctuate.

This techniques compromises important benefits over domain type method such as finite element and finite differences. One of the advantages is that with panels one only has to define the surface of the body, whereas with domain methods it is essential to discretize the entire region of the flow field. Moreover, this method is well–suited to problems with an infinite domain. The volume of input data for a panel method is therefore meaningfully less than for a field method, which is very important advantage in practice, as many hours can be consumed in formulating and proving the data for finite element or finite difference programs.

The panel methods are offered under different names such as “Boundary Element Method”, “Surface Singularity Methods”, “Boundary Integral Methods”, or “Boundary Integral equation Methods”. This method has been positively applied in numerous fields, for example, elasticity, potential theory, elasto-statics and elasto-dynamics.

The integral equation of Panel methods of higher order were also used for the flow field calculation by Hess and Simth [1967]. In the historical age, a direct panel method for potential flow problems was applied only once by Morino et al [1975]. Their methods take benefits of the distribution of constant potential over the quadrilateral elements of the body surface.

The velocity at the centroid of each element is to be found with the use of shape function. Consequently it is essential to apply the indirect panel method to calculate the ideal flow solution around two-dimensional bodies.

The equation of indirect panel method can be derived in the past by many authors. (Lamb [1932], Milne-Thomson [1968], Shah [2008], Brebbia and Walker [1980]). The direct and Indirect methods have been used in the past for flow field calculations around arbitrary bodies (Morino et al [1975], Hess and Smith[1967], Luminita et al [2008], Mushtaq et al [2009], Muhammad[2011]; Mushtaq and Shah [2010a, 2010b and 2012], Mushtaq [2011]). In this research paper, the indirect panel method has been used for the solution of ideal flow around a joukowski aerofoil using constant approach with doublet distribution alone.

Flow past a Joukowski Aerofoil: Consider a ideal flow around a Joukowski Aerofoil and let the onset flow be the uniform stream with velocity U in the positive direction of x-axis as shown in the figure 1.

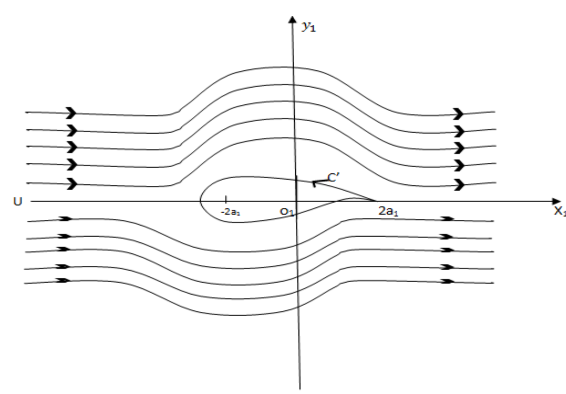

Figure 1: Flow past a joukowski aerofoil: Chow[1979], Mushtaq[2011], Mushtaq and Shah [2012] given the modulus of the exact velocity distribution over the boundary of a Joukowski Aerofoil is

$$V = U\left|\frac{1-\frac{r^2}{(z_1-z_2)^2}+\frac{2i_1c_1}{(z_1-z_2)}}{1-(\frac{a_1}{z_1})^2}\right| \qquad (1)$$

Where r = radius of the circular cylinder (C.C.)
=Joukowski transformation constant in equation (1)

$_V = x_1, \quad z_2 = b_1, \qquad b_1 = a_1 - \sqrt{r}$

Thus equation (1) can be written in Cartesian coordinates as

$$= U\sqrt{\frac{[\{(x_1-b_1)^2+(y_1-c_1)^2\}^2 - r^2\{(x_1-b_1)^2-(y_1-c_1)^2\}+2c_1(y_1-c_1)\{(x_1-b_1)^2+(y_1-c_1)^2]^2 + [2c_1(x_1-b_1)\{(x_1-b_1)^2+(y_1-c_1)^2+2r_1^2(x_1-b_1)(y_1-c_1)]^2}{[(x_1-b_1)^2+(y_1-c_1)^2]^2}}$$

$$\times \frac{\sqrt{[(x_1^2+y_1^2)^2 - a_1^2(x_1^2-y_1^2)^2+4a_1^4x_1^2y_1^2}}{(x_1^2-y_1^2-a_1^2)^2+4x_1^2y_1^2} \qquad (2)$$

Boundary Conditions: Since the boundary conditions which are essential for the solution of the boundary of Joukowski Aerofoil for an irrotational and steady flows are given as (Mushtaq and Shah [2010a,2010b, and 2012], Mushtaq [2011]).

$$\frac{\partial \phi_{j.a}}{\partial n} = U\left(-\frac{(y_c - y_d)}{\sqrt{(x_c - x_d)^2 + (y_c - y_d)^2}}\right) \qquad (4)$$

Process of Discretization: Panel method is used to resolve 2-dimensional interior and exterior ideal flow problem. The coordinates of the end points of the panel method for the discretization of the surface of the Jouukowski Aerofoil can be obtained within the computer program as given.

Distributing the boundary of C.C. into "n" panels by applying the formula (Muhammad [2011], Mushtaq et al[2009], Mushtaq and Shah [2010a,2010b, and 2012], Mushtaq [2011]).

$$\theta_m = [(n-3) - 2m]\frac{\pi}{\infty} \qquad (5) \qquad m = 1,2,3,\dots,n$$

And the extreme points of "n" panels of C.C. are

$$\xi_m = -b_1 + r\cos\theta_m$$

$$\eta_m = c_1 + r\sin\theta_m$$

Thus the end points of the Joukowski [a] using the joukowski transformation are given by

$$z_m = \zeta_m + \frac{a^2}{\zeta_m}$$

$$x_m + iy_m = (\xi_m + i\eta_m) + \frac{a^2}{\xi_m + i\eta_m}$$

$$= (\xi_m + i\eta_m) + \frac{a^2(\xi_m - i\eta_m)}{{\xi_m}^2 + {\eta_m}^2}$$

Equating real and imaginary parts, we get

$$\left.\begin{aligned} x_m &= \xi_m\left(1 + \frac{a^2}{{\xi_m}^2 + {\eta_m}^2}\right) \quad \text{where } m = 1,2,3,\dots,n \\ y_m &= \eta_m\left(1 - \frac{a^2}{{\xi_m}^2 + {\eta_m}^2}\right) \quad \text{where } m = 1,2,3,\dots,n \end{aligned}\right\} \qquad (6)$$

Matrix Formulation: Since for an interior flow problem, the equation of indirect panel method for the doublet distribution alone is:

$$-c_l\phi_l + \frac{1}{2\pi}\int_{\eta_1-1}\phi\frac{\partial}{\partial n}\left(\log\frac{1}{r}\right)d\eta_1 + \phi_\infty = -(\phi_{u.s})_l \qquad (7)$$

If the boundary of the body is discretized into "n"elements , then the equation (7) will be

$$-c_l\phi_l + \sum_{m=1}^{n}[\frac{1}{2\pi}\int_{\eta_1-1}\phi\frac{\partial}{\partial n}\log(\frac{1}{r})d\eta_1] = -(\phi_{u.s})_l \qquad (8)$$

Here "bethe length of the element "m" in terms of "l".

In constant element case we assumed the value of "be constant at each point and equal to the value at the mid-node of the element. In this case the number of nodes is the same as the number of elements "n". In this case "is fixed as the boundary condition on each element of the variable. We study the " " is constant, then we will take out of the integral. Then the equation will be

$$-c_l\phi_l + \sum_{m=1}^{n}[\frac{1}{2\pi}\int_{\eta_1-1}\frac{\partial}{\partial n}(log\frac{1}{r})d\eta_1]\phi_m = -(\phi_{u.s})_l \qquad (9)$$

Applying the equation (9) for a specific node "l" and the integrals $\frac{1}{2\pi}\int_{\eta_1-1}\frac{\partial}{\partial n}\Big(log$ transmit the node "l" with the element "m" over which the integrals are calculated is used to denote these integrals.

From equation (9)

$$-c_l\phi_l + \sum_{m=1}^{n}\widehat{W}_{l,m}\phi_m = -(\phi_{u.s})_l \qquad (10)$$

The integrals in equation (10) are very tough to solve analytically and numerically. We use 4-point Gauss-Quadrature rule for the solution of the integrals. Now

Let

$$W_{l,m} = \left\{\begin{matrix} \widehat{W}_{l,m} & when\ l \neq m \\ \widehat{W}_{l,m} - c_l & when\ l = m \end{matrix}\right\} \qquad (11)$$

But for exterior flow problem the equation of indirect panel method for the doublet distribution can be stated as

$$-c_l\phi_l + \frac{1}{2\pi}\int_{\eta_1 - 1} \phi \frac{\partial}{\partial n}\left(log\frac{1}{r}\right) d\eta_1 + \phi_\infty = -(\phi_{us})_l \quad (12)$$

For exterior flow problem, assembling the equation (10) and equation (12) for the indirect boundary integral equation method can be inscribed as

Or

$$\sum_{m=1}^{n} W_{l,m}\phi_l + \phi_\infty = -(\phi_{us})_l \quad (14)$$

When all nodes are in consideration, then equation (14) yields an $R \times ($ system of equations which can be written in matrix form in the case of constant and linear elements as

$$[W][\vec{H}] = [\vec{F}] \quad (15)$$

Here is the matrix coefficient, is a vector of unknown total potentials "and on the R.H.S is a known vector whose elements are the negative of the values of the velocity potentials of the uniform stream at the nodes on the origion of the body. It is clear that in equation (15) has (R+1) unknowns $\phi_1, \phi_2, \phi_3, \dots,$.

At particular point the value of "must be specific to solve the system of equations. For correctness " is designated as zero. This R (R+1) system shrinks to an R system of equations which can be resolved as before but now the diagonal coefficients will be found by

$$[W_{ll}] = -\sum_{\substack{m=1\\m=l}}^{R} W_{ml} - 1 \quad (16)$$

Thus the velocity in the mid of two nodes over the boundary, can then be calculated by using the given formula

$$\vec{V} = \frac{\phi_{m+1} - \phi_m}{\text{Length from node m to m} + 1} \quad (17)$$

Hence the given tables 1-3 show the comp. and exact vel. distribution comparison over the surface of a Joukowski aerofoil using doublet distribution alone with constant approach.

Table-1 Comparison of computed and exact velocity over the surface of a Joukowski aerofoil using 8 indirect constant panels with doublet distribution alone.

ELEMENT	X	Y	R	Comp. VEL.	EXACT VEL.
1	-12.39	2.40	12.62	.82892E+00	.79656E+00
2	-9.16	5.63	10.75	.20011E+01	.18792E+01
3	-4.58	5.63	7.26	.20008E+01	.18796E+01
4	-1.35	2.39	2.75	.82840E+00	.79415E+00
5	-1.35	-2.17	2.56	.82829E+00	.73108E+00
6	-4.58	-5.41	7.09	.20008E+01	.18167E+01
7	-9.16	-5.41	10.64	.20011E+01	.18163E+01
8	-12.39	-2.18	12.58	.82893E+00	.73371E+00

Table-2 Comparison of computed and exact velocity over the surface of a Joukowski aerofoil using 16 indirect constant panels with doublet distribution alone.

ELEMENT	X	Y	R	Comp. Vel.	Exact Vel.
1	-13.47	1.42	13.55	.39789E+00	.42147E+00
2	-12.47	3.85	13.05	.11331E+01	.11423E+01
3	-10.61	5.71	12.05	.16957E+01	.16942E+01
4	-8.18	6.71	10.59	.20002E+01	.19932E+01
5	-5.56	6.71	8.71	.20000E+01	.19934E+01
6	-3.13	5.71	6.51	.16952E+01	.16945E+01
7	-1.27	3.85	4.05	.11316E+01	.11414E+01
8	-.27	1.42	1.44	.39720E+00	.41532E+00
9	-.27	-1.20	1.23	.39678E+00	.35134E+00
10	-1.27	-3.62	3.84	.11315E+01	.10785E+01
11	-3.13	-5.49	6.32	.16952E+01	.16317E+01
12	-5.56	-6.49	8.55	.20001E+01	.19306E+01
13	-8.18	-6.49	10.45	.20002E+01	.19303E+01
14	-10.61	-5.49	11.95	.16958E+01	.16314E+01
15	-12.47	-3.63	12.99	.11331E+01	.10794E+01
16	-13.47	-1.20	13.53	.39789E+00	.35863E+00

Table-3 Comparison of computed and exact velocity over the surface of a Joukowski aerofoil using 32 indirect constant panels with doublet distribution alone.

ELEMENT	X	Y	R	Comp.Vel.	Exact Vel.
1	-13.77	.79	13.79	.19701E+00	.22739E+00
2	-13.50	2.12	13.67	.58344E+00	.61181E+00
3	-12.98	3.38	13.42	.94747E+00	.97396E+00
4	-12.23	4.51	13.03	.12750E+01	.12999E+01
5	-11.27	5.47	12.53	.15537E+01	.15772E+01
6	-10.14	6.22	11.90	.17725E+01	.17952E+01
7	-8.88	6.74	11.15	.19233E+01	.19454E+01
8	-7.55	7.01	10.30	.20001E+01	.20221E+01
9	-6.19	7.01	9.35	.20000E+01	.20222E+01
10	-4.86	6.74	8.31	.19230E+01	.19457E+01
11	-3.60	6.22	7.19	.17721E+01	.17956E+01
12	-2.47	5.47	6.00	.15530E+01	.15774E+01
13	-1.51	4.50	4.75	.12741E+01	.12996E+01
14	-.76	3.37	3.46	.94603E+00	.97246E+00
15	-.23	2.11	2.13	.58045E+00	.60803E+00
16	.03	.78	.78	.19648E+00	.21568E+00
17	.03	-.56	.56	.19508E+00	.14877E+00
18	-.23	-1.89	1.90	.57955E+00	.54472E+00
19	-.76	-3.15	3.24	.94589E+00	.90949E+00
20	-1.51	-4.28	4.54	.12741E+01	.12367E+01
21	-2.47	-5.25	5.80	.15530E+01	.15146E+01
22	-3.60	-6.00	7.00	.17721E+01	.17327E+01
23	-4.86	-6.52	8.13	.19231E+01	.18829E+01
24	-6.19	-6.79	9.19	.20000E+01	.19593E+01
25	-7.55	-6.79	10.15	.20001E+01	.19592E+01
26	-8.88	-6.52	11.02	.19233E+01	.18825E+01
27	-10.14	-6.00	11.78	.17725E+01	.17323E+01
28	-11.27	-5.25	12.43	.15537E+01	.15144E+01
29	-12.23	-4.29	12.96	.12751E+01	.12371E+01
30	-12.98	-3.16	13.36	.94746E+00	.91111E+00
31	-13.50	-1.90	13.64	.58345E+00	.54896E+00
32	-13.77	-.57	13.78	.19701E+00	.16454E+00

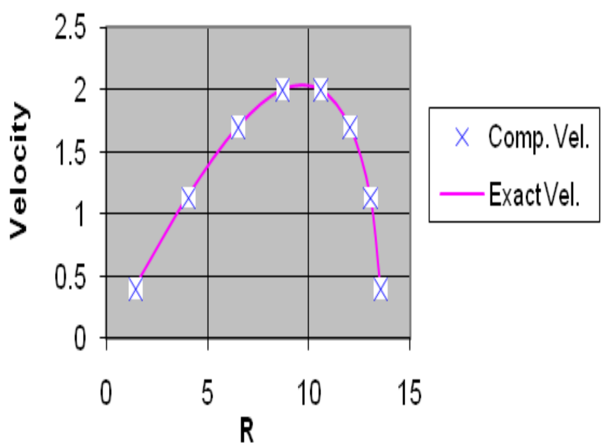

Figure 2: Comparison of computed and exact velocity over the surface of a Joukowski aerofoil using upper 8 of 16 indirect constant panels with doublet distribution alone.

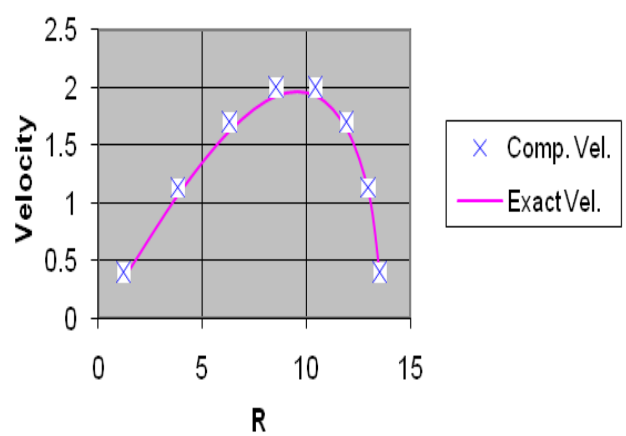

Figure 3: Comparison of computed and exact velocity over the surface of a Joukowski aerofoil using lower 8 of 16 indirect constant panels with doublet distribution alone.

Conclusion: An indirect panel method has been used for the computation of ideal flow(i.e. velocity distribution) around the surface of a Joukowski aerofoil using doublet distribution alone with constant approach. The computed flow velocities obtained using this technique is compared with the analytical velocities for ideal flow around the surface of a Joukowski aerofoil. The results obtained with the indirect panel method for the ideal flow field calculations are nearly close with the analytical results for the body under consideration.

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