VIBRATION ANALYSIS OF THREE-LAYERED FUNCTIONALLY GRADED CYLINDRICAL SHELLS WITH ISOTROPIC MIDDLE LAYER RESTING ON WINKLER AND PASTERNAK FOUNDATIONS

A. G. Shah, M. N. Naeem[1], A. Ali[1], T. Mahmood[2], S. H. Arshad[3]

Department of mathematics, Govt. Post Graduate College Jhang, Jhang 35200, Pakistan
[1]Department of mathematics, G. C. University Faisalabad, Faisalabad 38000, Pakistan
[2]Department of mathematics, the Islamia University of Bahawalpur, Bahawalpur 63100, Pakistan
[3]Department of Civil Engineering, Iqra National University Peshawar, Peshawar 25000, Pakistan
Corresponding Author's Email: aghafarshah@yahoo.com

ABSTRACT: Vibration characteristics of a cylindrical shell composed of three layers are investigated. The inner and outer layers of cylindrical shell are functionally graded materials whereas middle layer is of isotropic material. Love shell equations are used to study the vibration problem. The expressions for moduli of the Winkler and Pasternak foundations are combined with the shell dynamical equations. The wave propagation technique is used to solve the present shell problem. A number of comparisons of numerical results are performed to check the validity and accuracy of the present approach.

Key words: Vibration, Functionally graded cylindrical shell, Love's theory, Wave propagation method, Winkler and Pasternak foundations.

INTRODUCTION

The shell vibration problem is an extensively studied area of research in the structural dynamics. Numerical solutions of shell vibration problem started to come out in thirties of twentieth century and were presented by (Flügge, 1943). (Leissa, 1993) compared various thin shell theories and introduced different eighth-order system of differential equations in terms of different differential operators. (Blevins, 1979) gave a detailed review of formulas for shell structures. (Forsberg, 1964) carried out an extensive numerical analysis to demonstrate the significance of the effects of boundary conditions on the free vibration characteristics of circular cylindrical shells. (Goldman, 1974) evaluated the natural frequencies as well as mode shapes of thin-walled cylindrical shells. He used exponential function for axial modal dependence with clamped-clamped end conditions. (Loy *et al*. 1998) investigated the vibrations of functionally graded material cylindrical shells, made up of FG material composed of stainless steel and nickel. The purpose of work was to examine natural frequencies, influence of the constituent volume fractions and effects of configurations of constituent materials on their frequencies. (Pardhan *et al*. 2000) studied the vibrations of functionally graded material (FGM) cylindrical shell structured from stainless steel and zirconia. Influence of boundary conditions and volume fractions on natural frequencies of FGM cylindrical shell was studied. (Zhang *et al*. 2000) analyzed the vibrations of cylindrical shells employing wave propagation approach. Comparison of numerical results obtained by using the wave propagation approach and numerical finite element method were executed. (Naeem and Sharma, 2000) have employed Rayleigh-Ritz method to predict natural frequencies for thin cylindrical shells using Ritz polynomial for axial model dependence.

(Najafizadeh and Isvandzibaei, 2007) studied the vibrations of thin-walled cylindrical shells with ring supports composed of functionally graded material comprised of stainless steel and nickel. (Shao and Ma, 2007) investigated free vibration analysis of laminated cylindrical shells with arbitrary classical end conditions. Strain displacement relations from Love's shell theory were used in the study. (Iqbal *et al*. 2009) applied wave propagation approach to analyze vibrations of functionally graded material circular cylindrical shells. This methodology was very easy to apply. Axial model dependence was carried out by exponential functions. (Arshad *et al*. 2010) studied vibration of bi-layered cylindrical shells with layers of different materials. One layer was made of functionally graded material and the other layer of isotropic material. Frequencies were evaluated for long, short, thick and thin cylindrical shells by varying non dimensional geometrical parameters, length-to-radius and thickness-to- radius ratios for a simply supported boundary condition. Also (Arshad *et al*. 2010) investigated vibration analysis of bi-layered functionally graded cylindrical shells. In this case, both layers are composed of functionally graded materials and the thickness of shell layers is considered to be equal and constant. (Shah *et al*. 2010) have studied vibrations of functionally graded cylindrical shells based on elastic foundations. They amended the equations of functionally graded cylindrical shells by inducting the modulii of the Winkler and Pasternak foundations. Recently (Shah *et al*. 2010) presented vibration characteristics of cylindrical shells which was filled with fluid and was put on the elastic foundations. (Naeem *et al*. 2010) studied the vibration frequency characteristics of functionally graded cylindrical shells using the generalized differential quadrature method. The method was founded on the approximation of the derivatives of the unknown functions involved in differential equations at the mesh points of the solution domain. It was a sophisticated technique that gives accurate and robust results. A number of comparisons were done to check the effectiveness, robustness and accuracy of the presented method.

Formulation of the Shell Problem: A thin- walled cylindrical shell with the geometrical dimensions: length L, thickness h and mean radius R is shown in Fig. 1. An orthogonal coordinate system (x, φ, z) is considered to be at the mid surface of the shell whereas x, φ and z stand for the axial, circumferential and radial coordinates respectively. Young's modulus E, the Poisson ratio ν and the mass density ρ denote material parameters of the shell. The axial, circumferential and radial displacement deformations are represented by u(x, φ, t), v(x, φ, t) and w(x, φ, t) respectively with regard to the shell middle surface.

The equations of shell motion from the Love shell theory are in a differential operator form as:

$$L_{11}u + L_{12}v + L_{13}w = \rho_t \frac{\partial^2 u}{\partial t^2} \quad (1)$$

$$L_{21}u + L_{22}v + L_{23}w = \rho_t \frac{\partial^2 v}{\partial t^2} \quad (2)$$

$$L_{31}u + L_{32}v + L_{33}w = \rho_t \frac{\partial^2 w}{\partial t^2} + Kw - G\nabla^2 w \quad (3)$$

where $L_{ij}(i, j = 1,2,3)$ state the differential operators with regard to x and φ and are given as

$$L_{11} = A_{11}\frac{\partial^2}{\partial x^2} + \frac{A_{66}}{R^2}\frac{\partial^2}{\partial \varphi^2},$$

$$L_{12} = \frac{(A_{12}+A_{66})}{R}\frac{\partial^2}{\partial x \partial \varphi} + \frac{(B_{12}+2B_{66})}{R^2}\frac{\partial^2}{\partial x \partial \varphi},$$

$$L_{13} = \frac{A_{12}}{R}\frac{\partial}{\partial x} - B_{11}\frac{\partial^3}{\partial x^3} - \frac{(B_{12}+2B_{66})}{R^2}\frac{\partial^3}{\partial x \partial \varphi^2},$$

$$L_{21} = \frac{(A_{12}+A_{66})}{R}\frac{\partial^2}{\partial x \partial \varphi} + \frac{(B_{12}+B_{66})}{R^2}\frac{\partial^2}{\partial x \partial \varphi},$$

$$L_{22} = \left(A_{66} + \frac{3B_{66}}{R} + \frac{2D_{66}}{R^2}\right)\frac{\partial^2}{\partial x^2} + \left(\frac{A_{22}}{R^2} + \frac{2B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^2}{\partial \varphi^2},$$

$$L_{23} = \left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}\right)\frac{\partial}{\partial \varphi} - \left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^3}{\partial \varphi^3} - \left(\frac{B_{12}+2B_{66}}{R} + \frac{D_{12}+2D_{66}}{R^2}\right)\frac{\partial^3}{\partial x^2 \partial \varphi},$$

$$L_{31} = -\frac{A_{12}}{R}\frac{\partial}{\partial x} + B_{11}\frac{\partial^3}{\partial x^3} + \frac{(B_{12}+2B_{66})}{R^2}\frac{\partial^3}{\partial x \partial \varphi^2},$$

$$L_{32} = -\left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}\right)\frac{\partial}{\partial \varphi} + \left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^3}{\partial \varphi^3} + \left(\frac{B_{12}+2B_{66}}{R} + \frac{D_{12}+4D_{66}}{R^2}\right)\frac{\partial^3}{\partial x^2 \partial \varphi},$$

$$L_{33} = -\frac{A_{22}}{R^2} + \frac{2B_{12}}{R}\frac{\partial^2}{\partial x^2} + \frac{2B_{22}}{R^3}\frac{\partial^2}{\partial \varphi^2} - D_{11}\frac{\partial^4}{\partial x^4} - 2\frac{D_{12}+2D_{66}}{R^2}\frac{\partial^4}{\partial x^2 \partial \varphi^2} - \frac{D_{22}}{R^4}\frac{\partial^4}{\partial \varphi^4}, \quad (4)$$

where G stands for Pasternak elastic foundation and K for the Winkler foundation modulus. The expression for the differential operator ∇^2 is:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2}{\partial \varphi^2} \quad (5)$$

The following modal displacement shape functions are adopted to separate the time and space variables

$$u(x, \varphi, t) = A\cos(n\varphi)e^{i(\omega t - k_m x)}$$

$$v(x, \varphi, t) = B\sin(n\varphi)e^{i(\omega t - k_m x)}$$

$$w(x, \varphi, t) = C\cos(n\varphi)e^{i(\omega t - k_m x)} \quad (6)$$

in the longitudinal, circumferential and transverse directions respectively. The constants A, B and C are the amplitudes of vibrations in the x, φ and z directions respectively, n is the number of circumferential waves and k_m stands for axial wave number that is associated with a boundary condition is axial wave number given for four types of boundary conditions in Table 1:

Table 1

Boundary conditions	Wave numbers
Simply supported - simply supported	$k_m = m\pi/L$
Clamped – clamped	$k_m = (2m+1)\pi/2L$
Clamped - simply supported	$k_m = (4m+1)\pi/4L$
Clamped – free	$k_m = (2m-1)\pi/2L$

These axial wave numbers k_m are selected to satisfy boundary conditions at both edges of a cylindrical shell. ω denotes the natural angular frequency for the cylindrical shell.

Making substitution for the displacement functions u, v and w from the expressions (6) in system of equations (1-3), the shell algebraic equations are written as:

$$c_{11}A + c_{12}B + c_{13}C = \omega^2\rho_t A \quad (7)$$

$$c_{21}A + c_{22}B + c_{23}C = \omega^2\rho_t B \quad (8)$$

$$c_{31}A + c_{32}B + c_{33}C + Kw + G(k_m^2 + e_{313}n^2) = \omega^2\rho_t C \quad (9)$$

After the arrangement of terms, the algebraic simultaneous equations (7)-(9) are written in matrix notation as:

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ -c_{21} & c_{22} & c_{23} \\ -c_{13} & c_{23} & c_{33} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \omega^2 \rho_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} \quad (10)$$

where $C_{ij}\ (i, j = 1,2,3)$ are coefficients of stiffness matrix and their values are given below:

$$C_{11} = k_m^2 A_{11} + n^2 \frac{A_{66}}{R^2}$$

$$C_{12} = ink_m \frac{A_{12} + A_{66}}{R} + \frac{2B_{66} + B_{12}}{R^2}$$

$$C_{13} = ik_m \left\{ \frac{A_{12}}{R} - n^2 \frac{2B_{66} - B_{12}}{R^2} + k_m^2 B_{11} \right\}$$

$$C_{21} = -ink_m \frac{A_{12} + A_{66}}{R} + \frac{B_{66} + B_{12}}{R^2}$$

$$C_{22} = k_m^2 \left\{ A_{66} + n^2 \frac{A_{66}}{R^2} + 3\frac{B_{66}}{R} + 2\frac{D_{66}}{R^2} \right\} + n^2 \left\{ \frac{A_{22}}{R^2} + \frac{2B_{22}}{R^3} + \frac{D_{22}}{R^4} \right\}$$

$$C_{23} = nk_m^2 \left\{ \frac{2B_{66} + B_{12}}{R} + \frac{2D_{66} + D_{12}}{R^2} \right\} + n \left\{ \frac{A_{22}}{R^2} + \frac{B_{22}}{R^3} \right\} + n^3 \left\{ \frac{B_{22}}{R^3} + \frac{D_{22}}{R^4} \right\}$$

$$C_{31} = -ik_m \left\{ \frac{A_{12}}{R} - n^2 \frac{2B_{66} - B_{12}}{R^2} + k_m^2 B_{11} \right\}$$

$$C_{23} = nk_m^2 \left\{ \frac{2B_{66} + B_{12}}{R} + \frac{2D_{66} + D_{12}}{R^2} \right\} + n \left\{ \frac{A_{22}}{R^2} + \frac{B_{22}}{R^3} \right\} + n^3 \left\{ \frac{B_{22}}{R^3} + \frac{D_{22}}{R^4} \right\}$$

$$C_{33} = -\frac{A_{22}}{R^2} + 2n^2 \frac{B_{22}}{R^3} + 2k_m^2 \frac{B_{12}}{R} + n^4 \frac{D_{22}}{R^4} + 2nk_m^2 \frac{2D_{66} + D_{12}}{R^2} + k_m^4 D_{11} + K + G\left\{ k_m^2 + \frac{n^2}{R^2} \right\}$$

(11)

where A_{ij} , B_{ij} and $D_{ij}(i, j = 1,2 \text{ and } 6)$ stand for extensional, coupling and bending stiffness respectively and defined as:

$$\left. \begin{aligned} A_{ij} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij}\ dz \\ B_{ij} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij}\ z\, dz \\ D_{ij} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij}\ z^2\, dz \end{aligned} \right\} \quad (12)$$

ρ_t denotes the mass density per unit length and is defined as:

$$\rho_t = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho\, dz \quad (13)$$

For forming the shell frequency equation, the determinant of the matrix coefficients is vanished for non-trivial eigen-value of the shell frequency. Either the eigen-value problem obtained above is solved using computer program or the determinant is expanded and the frequencies equation is achieved in the form of polynomial equation involving ω^2.

RESULTS AND DISCUSSION

Numerous comparisons of numerical results for isotropic as well as functionally graded material cylindrical shells are carried out in order to examine validity, efficiency and accuracy of wave propagation approach. The numerical results for following four boundary conditions are calculated and compared with those found in the open literature.

i. simply supported - simply supported (SS-SS)
ii. clamped - clamped (C-C)
iii. clamped - simply supported (C-SS)
iv. clamped - free (C-F)

Simply Supported Cylindrical Shells: In Table 2, the non-dimensional frequency parameter $\Omega = \omega R\sqrt{(1-\nu^2)\rho / E}$ for an isotropic cylindrical shell is compared with those ones obtained by Naeem and Sharma. (1999).The simply supported boundary conditions are described on shell ends. In this comparison, the shell parameters, length-to-radius ratio and thickness-to-radius ratio are taken as: $L/R = 6$ and $h/R = 0.002$ respectively. The axial mode is assumed to be $m = 1$ and the circumferential wave numbers n are chosen from 1-10. Material properties of the shell are mass density (ρ), Poisson's ratio (ν) and Young's modulus (E) and their values are given as: $\rho = 7850$ kg/m^3, $\nu = 0.3$ and $E = 2.1 \times 10^{11}$ N/m^2.

Table 2 Comparison of frequency parameter $\Omega = \omega R\sqrt{(1-\nu^2)\rho / E}$ for a SS-SS cylindrical shell ($m = 1$, $L/R = 6$, $h/R = 0.002$, $\nu = 0.3$)

n	(Naeem and Sharma 1999)	Present	% Difference
1	0.140641	0.140642	0.000
2	0.054323	0.054324	0.002
3	0.027074	0.027075	0.004
4	0.017776	0.017767	-0.051
5	0.017088	0.017074	-0.082
6	0.021303	0.021304	0.005
7	0.028089	0.028082	-0.025
8	0.036469	0.036470	0.003
9	0.046174	0.046171	-0.006

Clamped Cylindrical Shells: Table 3 shows comparison of non-dimensional frequency parameter Ω for a cylindrical shell executing vibration with clamped-clamped boundary conditions. The present values are compared with those in the reference: Zhang *et al*. (2001) for $L/R = 20$ and $h/R = 0.002$ where $m = 1$ and 2 and the circumferential wave numbers n are assumed to be 1-5.

Clamped-Simply Supported Cylindrical Shells: Table 4 displays a comparison of frequency parameters

$\Omega = \omega R\sqrt{(1-\nu^2)\rho/E}$ determined by the present method with ones determined by Naeem *et al*.(2009) for a clamped-simply supported cylindrical shell. A good agreement is obvious between the two sets of results.

Table 3 Comparison of values of the frequency parameter $\Omega = \omega R\sqrt{(1-\nu^2)\rho/E}$ for clamped-clamped (C-C) cylindrical shells ($\frac{L}{R} = 20, \frac{h}{R} = 0.002, \nu = 0.3$)

m	n	(Zhang *et al.* 2001b)	Present	% Difference
1	1	0.03487	0.03488	0.02868
	2	0.01176	0.01176	0.00000
	3	0.007083	0.007084	0.014118
	4	0.009016	0.009017	0.011091
	5	0.01377	0.01377	0.00000
2	1	0.08742	0.08742	0.00000
	2	0.03155	0.03155	0.00000
	3	0.01586	0.01586	0.00000
	4	0.01224	0.01224	0.00000
	5	0.01482	0.01482	0.00000

Table 4 Comparison of frequency parameter $\Omega = \omega R\sqrt{(1-\nu^2)\rho/E}$ for a clamped - simply supported cylindrical shells ($m = 1, L/R = 20, h/R = 0.002, \nu = 0.3$)

n	(Naeem *et al.* 2009)	Present	%Difference
1	0.024029	0.024721	2.88
2	0.008283	0.008282	-0.01
3	0.005844	0.005852	0.14
4	0.008705	0.008710	0.05
5	0.013678	0.013684	0.04
6	0.019973	0.019979	0.03
7	0.027459	0.027466	0.02
8	0.036111	0.036118	0.02
9	0.045984	0.045929	-0.12
10	0.056889	0.056897	0.01

Clamped-Free Cylindrical Shells: Table 5 illustrates frequency parameters Ω for a clamped-free shell. The analytical results were evaluated by Naeem *et al*. (2009) for axial wave number $m = 1$. There is once again an excellent agreement between the two sets of analytical results.

From the previous comparisons of numerical results for the shell problems, it is noticed that the method employed here is very efficient, valid, fast and provides accurate results.

Table: 5 Comparison of frequency parameter $\Omega = \omega R\sqrt{(1-\nu^2)\rho/E}$ for clamped-free (C-F) cylindrical shell ($m = 1$, $L = 254$ mm, $h = 1.5$ mm, $R = 32.25$ mm, $E = 207 \times 10^9$ N/m^2, $\upsilon = 0.28$, $\rho = 7.86 \times 10^3$ Kg/m^3)

n	(Naeem *et al.* 2009)	Present	% Difference
1	0.0348	0.0256	-26.44
2	0.0381	0.0373	-2.080
3	0.1022	0.1024	0.196
4	0.1954	0.1958	0.205

Three-Layered Cylindrical Shells: A number of results for the proposed three-layered functionally graded cylindrical shells are determined for various sets of material. The inner and outer layers of the shell are comprised of functionally graded material whereas the middle layer is assumed to be isotropic as shown in Fig.1 Material properties of shell are represented by Young's modulus (E), Poisson's ratio (ν) and mass density (ρ). In general, vibration characteristics are mostly influenced by Young's modulus. In this study, the Poisson's ratio is presumed to be constant for functionally graded materials whereas the Young's modulus depends on intrinsic thickness variable (z) as well as the Young's modulus of constituent materials forming functionally graded layers. Here two configurations of a cylindrical shell are considered to suggest with regard to the shell layer thickness. In first configuration, the thickness of each layer is supposed to be of h/3 while in the second configuration, the thickness of each of the inner and outer layers are of h/4 and that of middle layer is of h/2.

The stiffness moduli A_{ij}, B_{ij} and D_{ij} are modified according to the thickness of material layers when inner and outer layers are functionally graded and middle is isotropic as

$$A_{ij} = A_{ij}^{in(FG)} + A_{ij}^{m(isotropic)} + A_{ij}^{out(FG)}$$
$$B_{ij} = B_{ij}^{in(FG)} + B_{ij}^{m(isotropic)} + B_{ij}^{out(FG)}$$
$$D_{ij} = D_{ij}^{in(FG)} + D_{ij}^{m(isotropic)} + D_{ij}^{out(FG)} \quad (14)$$

where $i, j = 1,2,6$ and in(FG), m(isotropic) and out(FG) are associated with inner functionally graded, middle isotropic and outer functionally graded layers of cylindrical shell respectively. Here by considering the constituent material of stainless steel for isotropic layer and also the FG layers are structured from two kinds of materials, nickel and zirconia. In this way, four types of shells are obtained and are listed in the following Table 6

Table 6. Description of cylindrical shells with isotropic middle layer

Types of Shell	Inner FGM Layer	Isotropic	Outer FGM Layer
Type I	Nickel - Zirconia	Stainless Steel	Nickel - Zirconia
Type II	Zirconia - Nickel	Stainless Steel	Zirconia - Nickel
Type III	Nickel - Zirconia	Stainless Steel	Zirconia - Nickel
Type IV	Zirconia - Nickel	Stainless Steel	Nickel - Zirconia

Material properties of isotropic materials: Steel and Aluminium are given in Table 7 whereas the material properties of the constituent materials forming functionally graded layers are listed in Table 8

Table 7. Material properties of isotropic materials

Isotropic	E(N/m^2)	Poisson ratio(ν)	Density ρ (Kg/m^3)
Stainless Steel	68.95E+09	0.315	2.7145E+03
Aluminium	2.1E+11	0.28	7.8E+03

Table 8. Material properties of Nickel and Zirconia

FGM	E(N/m^2)	Poisson ratio(ν)	Density ρ (Kg/m^3)
Nickel	2.05098E+11	0.3100	8900
Zirconia	1.6806296E+11	0.297996	5700

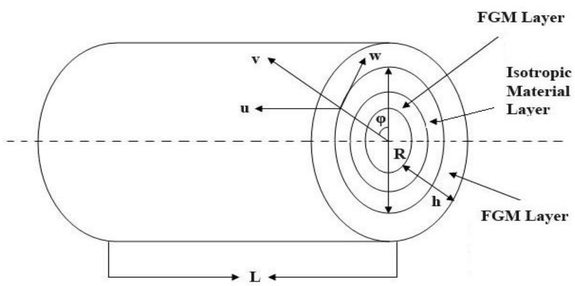

Fig.1 Geometry of three layered cylindrical shell

Frequency Analysis of cylindrical Shells: Tables 9-12, represent variations of natural frequencies (Hz) for Type I, II, III and IV cylindrical shells. Here the middle layer is supposed to be of isotropic and an outer and inner layer of the shell consists of functionally graded materials. Type I & III cylindrical shells have the approximate same frequency values whereas Type II & IV cylindrical shells have equal vibration frequencies. This shows that the interchange of the functionally graded materials forming the outer layers of the shell do not affect the frequency.

Table 9. Variation of natural frequencies (Hz) of a simply supported- simply supported three-layered (with isotropic middle layer) cylindrical shell on elastic foundations ($m = 1$, $p = 1$, $L = 0.41$m, $R = 0.3015$m, $h = 0.001$m, $\nu = 0.3$, $G = 1.5 \times 10^7$N-m, $K = 2.5 \times 10^7$ N-m)

n	Type I	Type II	Type III	Type IV
1	7696.81	7671.14	7696.67	7671.28
3	10990.1	10951.1	10989.9	10951.4
4	13269.6	13223.3	13269.3	13223.7
5	15720.2	15665.7	15719.8	15666.2

Table 10 Variation of natural frequencies (Hz) of a clamped-free three-layered (with isotropic middle layer) cylindrical shell on elastic foundations ($m = 1$,$p = 1$, $L = 0.41$m, $R = 0.3015$m, $h = 0.001$m, $\nu = 0.3$, $G = 1.5 \times 10^7$ N-m, $K = 2.5 \times 10^7$ N-m)

n	Type I	Type II	Type III	Type IV
1	10676.7	10645.4	10676.5	10645.6
2	11603.3	11562.0	11603.0	11562.3
3	13161.4	13114.5	13161.1	13114.8
4	15110.3	15057.2	15110.0	15057.6
5	17301.4	17241.7	17301.0	17241.6

Table 11. Variation of natural frequencies (Hz) of a clamped- simply supported three-layered (with isotropic middle layer) cylindrical shell on elastic foundations ($m = 1$,$p = 1$, $L = 0.41$m, $R = 0.3015$m, $h = 0.001$m, $\nu = 0.3$, $G = 1.5 \times 10^7$ N-m, $K = 2.5 \times 10^7$ N-m)

n	Type I	Type II	Type III	Type IV
1	9159.33	9130.67	9159.13	9130.88
2	10262.3	10225.2	10262.1	10225.4
3	12013.2	11970.3	12012.9	11970.6
4	14126.1	14076.6	14125.8	14077.0
5	16449.7	16392.6	16449.3	16393.0

Table 12 Variation of natural frequencies (Hz) of a clamped-free three-layered (with isotropic middle layer) cylindrical shell on elastic foundations ($m = 1$,$p = 1$, $L = 0.41$m, $R = 0.3015$m, $h = 0.001$m, $\nu = 0.3$, $G = 1.5 \times 10^7$N-m, $K = 2.5 \times 10^7$N-m)

n	Type I	Type II	Type III	Type IV
1	5159.96	5141.50	5159.89	5141.57
2	7082.46	7057.42	7082.30	7057.58
3	9471.44	9438.61	9471.19	9438.86
4	12040.0	11998.5	12039.7	11998.9
5	14694.3	14643.8	14693.9	14644.2

Fig.2 represents variation of natural frequency (Hz) for the three layered cylindrical shell with boundary conditions SS - SS, C - C, C - SS and C - F versus the axial mode $m = 1,2,3,4,5$. The frequency increases linearly with the axial mode (m) for each boundary condition. The frequency is the highest for clamped-clamped condition followed by clamped – simply supported, simply supported – simply supported, and clamped - free. This behaviour is due to the constraints involved in a condition.

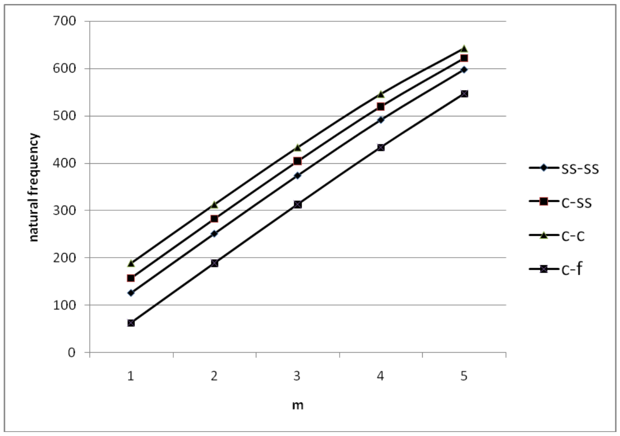

Fig. 2 Variations of natural frequencies (Hz) for the three layered cylindrical shell when $n = 0, \frac{L}{R} = 20, \frac{h}{R} = 0.002, p = 1$

Concluding Remarks: In this study, vibration characteristics of three-layered cylindrical shells are investigated. In the configuration, outer and inner layers are of functionally graded layers whereas the middle layer is isotropic. This forms a combination of isotropic and functionally graded layers. In a shell thickness direction, material composition of a functionally graded material is controlled by volume fraction law. Love shell dynamical equations are considered to describe the vibration problem. Pasternak and Winkler foundations are attached in the transverse direction. Wave propagation approach is applied to frame the shell frequency equation for a cylindrical shell in the eigen value form. MATLAB programs are written to extract vibration frequencies. It is observed that frequency increases with increasing values of circumferential wave number n. It is seen that natural frequencies of Type I cylindrical shell are identical to that of Type III cylindrical shell whereas natural frequencies of Type II and Type IV coincide with each other. It is concluded that frequency is the highest for clamped-clamped condition followed by clamped – simply supported, simply supported – simply supported, and clamped - free. This behaviour is due to the constraints involved in a condition.

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