MAGNETIC FIELD GENERATION DUE TO A SELF ACCELERATING DYNAMO IN AN EXTERNAL GRAVITATIONAL FIELD

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ABSTRACT: In this paper, the magnetic field generation due to a self-accelerating dynamo in an external gravitational field having calculated and attempt to reconcile it with the existence of gravity effect. (Schekochihin and Cowley, 2008) have proposed a model for the fast growth of magnetic field in galactic clusters using a self-accelerating dynamo. It is shown that the magnetic field amplification is quite large (up to10^8) factor over a very short period of time of 10^8yr. This must be so because the viscosity of the medium is determined not only by particular collision but also by plasma micro-instabilities by increasing the effective Reynold number. Here we extend this model and include the effects of the external gravitational field present in such system using the basic fluid equations for the ICM plasma. We modified these equations under the gravitational potential ✹ of $r,$ then the numerical results give us the graphs which combine the physical quantities are r radius, B gyroradius and dimensionless measure of the pressure anisotropies Δ.

Key words: Intracluster, Magnetic Field Generation, Gravitational Field/

INTRODUCTION

The dynamo theory proposes a mechanics by which a celestial body such as the earth generates a magnetic field. It is a process through which the rotating, convecting and electrically conducting fluids act to maintain a magnetic field. We use this theory to explain the presence of anomalously long-lived magnetic fields in astrophysical bodies (Schekochihin et al, 2005). We use the magneto hydrodynamics (MHD) equations to investigate how the fluid can continuously regenerates the magnetic field (Schuecker and Finoguenov, 2004; Schuecker and Bohringer, 2004). Many authors have been explained the idea of dynamo generated in the past (Brandenburg and Subramanian, 2005; Bruggen et al, 2005; Schekochihin et al, 2002, 2004, 2005; Vogt and Enβlin, 2003, 2005). The intracluster medium (ICM) is a unique environment for studying astrophysical turbulence because it contains huge amounts of magnetic energy (Carilli and Taylor, 2002; Dolag and Grasso, 2005; Fujita, 2005; Gaensler et al, 2004; Rebusco and Churazov, 2005; Takizwan, 2005). The plasma in cluster of galaxies is likely to be in a turbulence state. The MHD equations for the fluid velocity $\mathbf{v}$ and magnetic field $\mathbf{B}$ have the following general form

$$\rho\frac{d\mathbf{v}}{dt}=-\nabla\left(P_\perp+\frac{B^2}{8\pi}\right)+\nabla\cdot\left[\hat{\mathbf{b}}\hat{\mathbf{b}}\left(P_\perp-P_\parallel\right)\right]+\frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi}, \tag{1}$$

$$\frac{d\mathbf{B}}{dt}=\mathbf{B}\cdot\nabla\mathbf{v}-\mathbf{B}\nabla\cdot\mathbf{v}+\eta\nabla^2\mathbf{B}, \tag{2}$$

Where

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla, \tag{3}$$

is the material derivative and $\rho = m_i n,$ is known as mass density, $\hat{\mathbf{b}}=\frac{\mathbf{B}}{B}$. $P_\parallel$ and $P_\perp$ are the plasma pressures parallel and perpendicular to the magnetic field. Eqn (1) is valid provided the ions are magnetized i.e., $\rho_i \ll \lambda_{mfp}$ (Schekochihin and Cowley, 2002, 2005, 2008). The fundamental property of charged particles moving in a magnetic field is the conservation of the first adiabatic invariant (Schekochihin and Cowley, 2004, 2008). Then

$$\frac{1}{P_\perp}\frac{dP_\perp}{dt}=\frac{1}{B}\frac{dB}{dt}-\nu_i\frac{P_\perp-P_\parallel}{P_\perp}, \tag{4}$$

Where from (Schekochihin and Cowley, 2008) we get

$$\frac{1}{B}\frac{dB}{dt}=\nu_i\Delta, \tag{5}$$

Where $\Delta = \dfrac{\left(P_{\perp} - P_{\parallel}\right)}{P_{\perp}},$ is known as the dimensionless measure of the pressure anisotropies and the change of the magnetic field strength can be expressed in terms of the fluid velocity $\mathbf{v}$, (Fabian and Sanders, 2003; Schekochihin and Cowley, 2008).

$$\frac{1}{B}\frac{dB}{dt} = \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\mathbf{v} - \nabla.\mathbf{v} = \frac{\mathbf{U}}{L}\mathrm{Re}^{\frac{1}{2}}, \quad (6)$$

Eqn (6) represents the turbulent rate of strain at the viscous scale be estimated. It is the parallel viscosity that matters here (Schekochihin et al, 2004, 2008).

$$\mathrm{Re} = \frac{\mathbf{U}L}{v_{thi}\lambda_{mfp}} = \frac{\mathbf{U}L}{v_{thi}^{2}} v_i, \quad (7)$$

We have from eqn (5) and eqn (6)

$$v_i \Delta = \frac{\mathbf{U}}{L}\mathrm{Re}^{\frac{1}{2}}, \quad (8)$$

And

$$\mathrm{Re}^{\frac{1}{2}} = \left(\frac{\mathbf{U}L}{v_{thi}^{2}} v_i\right)^{\frac{1}{2}}, \quad (9)$$

Finally, we get the result

$$\Delta = \left(\frac{\mathbf{U}}{v_{thi}}\right)^{2}\mathrm{Re}^{-\frac{1}{2}}, \quad (10)$$

This explosive growth of magnetic field possibility was explained by (Haugen and Brandenburg, 2012; Fujita, 2005; Schekochihin and Cowley, 2008). In this paper, we have analyzed the model of the self-accelerating cluster dynamo with the gravity effect Φ (Anjam, 2010).

A Self-accelerating cluster dynamic model with gravity effect: Here we consider the net effect v_e, of the instabilities is to change the effective collision frequency of ions:

$$v_e = v_i + v_{scatter}(B), \quad (11)$$

And

$$v_{scatter}(B) = \left(|\Delta| - \frac{2}{\beta}\right)^{\alpha}\Omega_i, \quad (12)$$

Where $\alpha = \alpha_1 + \alpha_2 > 0$ and Ω_i, from (Schekochihin and Cowley, 2008) replacing $v_i \to v_e$ in eqn (7) and in eqn (10) then we get

$$v_e = v_i + \left(|\Delta| - \frac{2}{\beta}\right)^{\alpha}\Omega_i, \quad (13)$$

Where $\left(v_i \sim \dfrac{v_{thi}}{\lambda_{mfp}}\right)$, and $\left(v_e = \dfrac{U^3}{(v_{thi})^2 L\Delta^2}\right)$, then the equation for the anisotropic ratio is

$$\frac{1}{\Delta^2} = \frac{1}{\Delta_o^2} + \frac{1}{\in}\left[|\Delta| - 2B^2\right]^{\alpha} B. \quad (14)$$

Where the magnetic field is in units of (Schekochihin and Cowley, 2008)

$$\Delta_o = \left[\left(\left(\frac{\mathbf{U}}{v_{thi}}\right)^{3}\frac{\lambda_{mfp}}{L}\right)\right]^{\frac{1}{2}} \sim 0.01, \quad (15)$$

$$\in = \left(\frac{\mathbf{U}}{v_{thi}}\right)^{3}\frac{\rho_{i,eq}}{L} = \frac{\rho_{i,eq}}{\lambda_{mfp}}\Delta_o^2 \sim 10^{-17}, \quad (16)$$

The result will not be very sensitive to the value of α. Here, we shall use $\alpha = \frac{3}{2}$, for specific estimates. The growth of the magnetic field satisfies (from eqn (6) and (Schekochihin and Cowley, 2008) with $v_i \to v_e$].

$$\frac{1}{B}\frac{dB}{dt} = \frac{1}{\Delta(B)}, \quad (17)$$

Where time is in units of (Schekochihin and Cowley, 2008) and $\Delta(B)$ is the solution of eqn (14).

Equation (14) and eqn (17) constitute our model of the growth. When we assume the gravitational potential Φ, (Anjam, 2010) in the model of the accelerating cluster dynamo, then the model takes the form by including the gravitational potential $\Phi = -\dfrac{k}{r},$ with magnetic field B. Then eqn (14) takes the form

$$\frac{1}{\Delta^2} = \frac{1}{\Delta_o^2} + \frac{1}{\in}\left[\Delta - 2(B+\Phi)^2\right]^{\alpha}(B+\Phi). \quad (18)$$

Thus eqn (18) solved analytically in three asymptotic regimes and then their numerical results will be discussed.

Exponential stage: If we take

$$\frac{1}{\Delta} \to 0, \qquad (19)$$

Then from eqn (18) we get

$$0 = \frac{\mathbf{1}}{\Delta_o^2} + \frac{1}{\in}\left[\Delta - 2(B+\Phi)^2\right]^{\alpha}(B+\Phi), \qquad (20)$$

And also by taking

$$\exp 2(B+\Phi)^2 \to 0, \qquad (21)$$

$$0 = \frac{\mathbf{1}}{\Delta_o^2} + \frac{1}{\in}(\Delta)^{\alpha}(B+\Phi), \qquad (22)$$

Such that $B \to B'$, and $\Delta \to \Delta_0$. Then

$$B' + \Phi = \frac{\in}{\Delta_o^{2+\alpha}}, \qquad (23)$$

or

$$B' = \frac{\in}{\Delta_o^{2+\alpha}} - \Phi = B_{eq}\frac{\rho_{i,eq}}{\lambda_{mfp}}\left[\left(\frac{v_{thi}}{\mathbf{U}}\right)^3 \frac{L}{\lambda_{mfp}}\right]^{\frac{\alpha}{2}} + \frac{k}{r}. \qquad (24)$$

Explosive stage: For $B \gg B'$, we may approximate the solution of eqn (18) by taking $\frac{1}{\Delta_o} \to 0,$

$$\frac{1}{\Delta^2} = 0 + \frac{1}{\in}\left[\Delta - 2(B+\Phi)^2\right]^{\alpha}(B+\Phi), \qquad (25)$$

Now for exponentially

$$2(B+\Phi)^2 \to 0, \qquad (26)$$

$$\frac{1}{\Delta^2} \approx \frac{1}{\in}(\Delta)^{\alpha}(B+\Phi), \qquad (27)$$

Then finally, we get the result

$$\Delta(B) \approx \left(\frac{\in}{B+\Phi}\right)^{\frac{1}{2+\alpha}}, \qquad (28)$$

$$\Delta(B) \approx \left(\frac{\in}{B-\frac{k}{r}}\right)^{\frac{1}{2+\alpha}}. \qquad (29)$$

Algebraic stage

When Δ stays just above $2(B+\Phi)^2$, then by computing the small correction eqn (18) give us

$$\frac{1}{4(B+\Phi)^4} = \frac{\mathbf{1}}{\Delta_o^2} + \frac{1}{\in}\left[\Delta - 2(B+\Phi)^2\right]^{\alpha}(B+\Phi), \qquad (30)$$

$$\frac{1}{4(B+\Phi)^4} = \frac{\mathbf{1}}{\Delta_o^2} + \frac{(B+\Phi)}{\in}\Delta^{\alpha}\left[1 - \frac{2(B+\Phi)^2}{\Delta}\right]^{\alpha}, \qquad (31)$$

$$\frac{1}{4(B+\Phi)^4} = \frac{\mathbf{1}}{\Delta_o^2} + \frac{(B+\Phi)}{\in}\Delta^{\alpha}\left[1 - \frac{2\alpha(B+\Phi)^2}{\Delta} + O(\Delta)\right], \qquad (32)$$

Leaving the square and higher order terms of Δ, and also by adding the correction term $2(B+\Phi)^2$, then we get the approximate solution of eqn (32) is

$$\Delta(B) \approx 2(B+\Phi)^2 + \left[\frac{\in}{(B+\Phi)}\left(\frac{1}{4(B+\Phi)^4} - \frac{\mathbf{1}}{\Delta_o^2}\right)\right]^{\frac{1}{\alpha}}, \qquad (33)$$

or

$$\Delta(B) \approx 2\left(B-\frac{k}{r}\right)^2 + \left[\frac{\in}{\left(B-\frac{k}{r}\right)}\left(\frac{1}{4\left(B-\frac{k}{r}\right)^4} - \frac{\mathbf{1}}{\Delta_o^2}\right)\right]^{\frac{1}{\alpha}}. \qquad (34)$$

Numerical Results and Discussion: In this section, we shall discuss the some graphs related to our work. We have divided this section into two parts. Firstly we have discuss the graphs of the model for different stages without the gravitational potential Φ and in the second part, we have discuss them with the effect of gravitational potential Φ.

Graphs of the self-accelerating cluster dynamo without gravity effect

The graph of the eqn (29) without gravity effect is given below

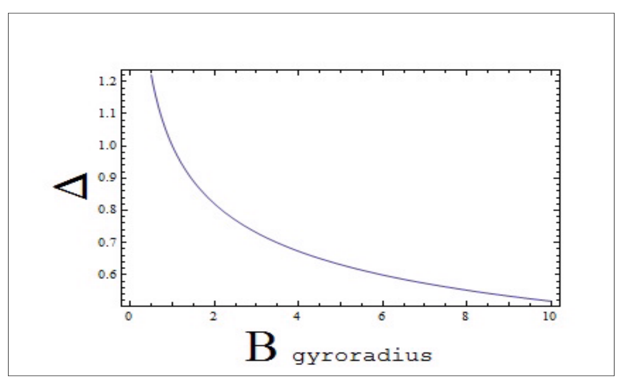

Fig.1 Graph of magnetic field and dimensionless measure of pressure anisotropies.

This is a graph of the explosive stage of the model without gravity effect. This graph shows the relation between magnetic field B gyroradius and dimensionless measure of the pressure anisotropies Δ. Thus the magnetic field B and the dimensionless measure of the pressure anisotropies Δ is inversely proportional to each other. The graph shows the discontinuity of the function. $B = 0,$ shows the asymptote of the graph.

The graph of the eqn (34) without gravity effect is given below.

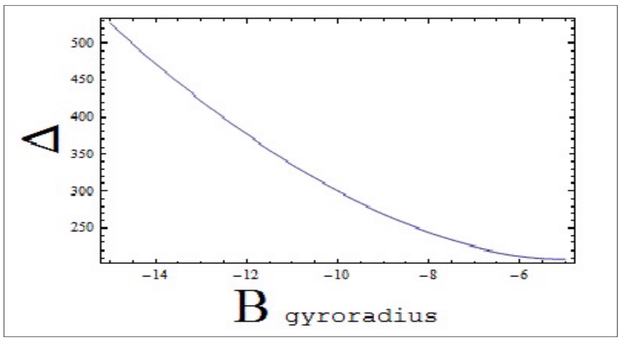

Fig.2 Graph of magnetic field and dimensionless measure of pressure anisotropies.

The graph shows the relation between magnetic field B gyroradius and dimensionless measure of the pressure anisotropies Δ · This graph is also shows that the magnetic field B and the dimensionless measure of the pressure anisotropies Δ is inversely proportional to each other. The graph shows the discontinuity of the function.

Graphs of the self-accelerating cluster dynamo with gravity effect: The graph of the eqn (24) with the gravity effect is given below.

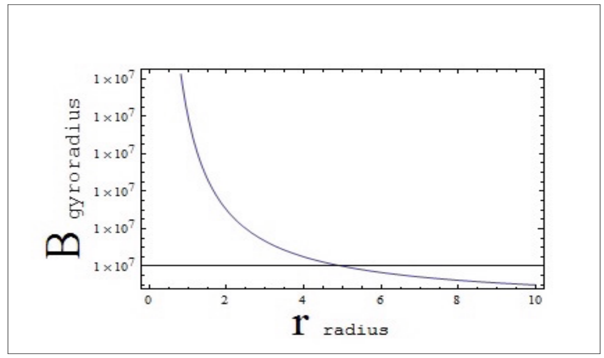

Fig.3 Graph between radius and magnetic field.

This is a graph for exponential stage of self-accelerating cluster dynamo with the gravity effect. This shows the relation between radius r and magnetic field B gyroradius. This is a discontinuous graph. Thus the magnetic field and the radius is inversely proportional to each other.

The graph of eqn (29) with gravity effect is given below.

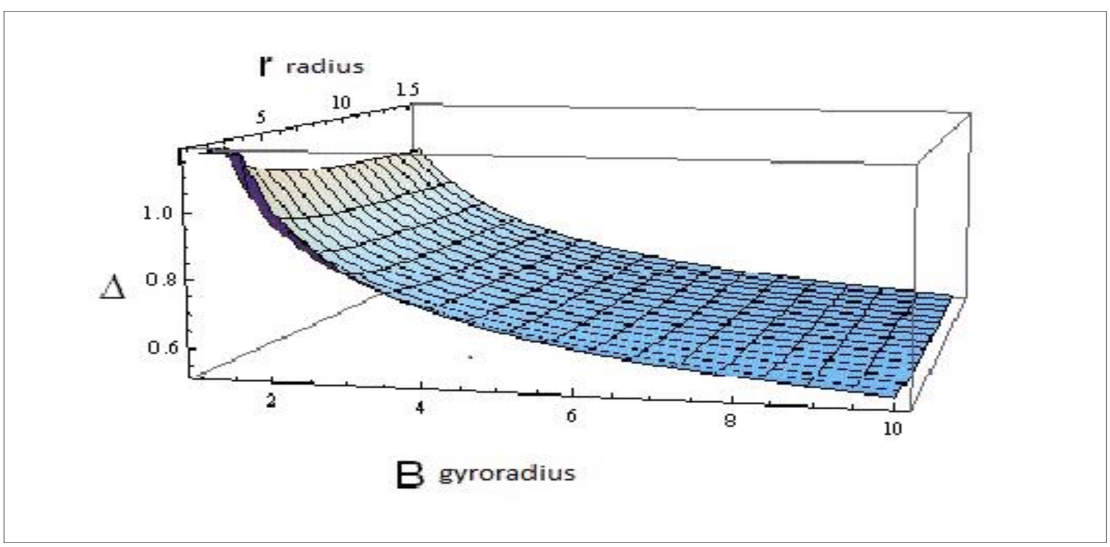

Fig.4 Graph between magnetic field, radius and dimensionless measure of pressure anisotropies.

This shows the relation between radius r, magnetic field B gyroradius and dimensionless measure of the pressure anisotropies Δ. This 3D graph combine the physical quantities are r radius, B gyroradius and dimensionless measure of the pressure anisotropies Δ. By increasing the value of magnetic field, the value of Δ and r are decreased. Also this graph goes to infinity.

The graph of eqn (34) with gravity effect is given below.

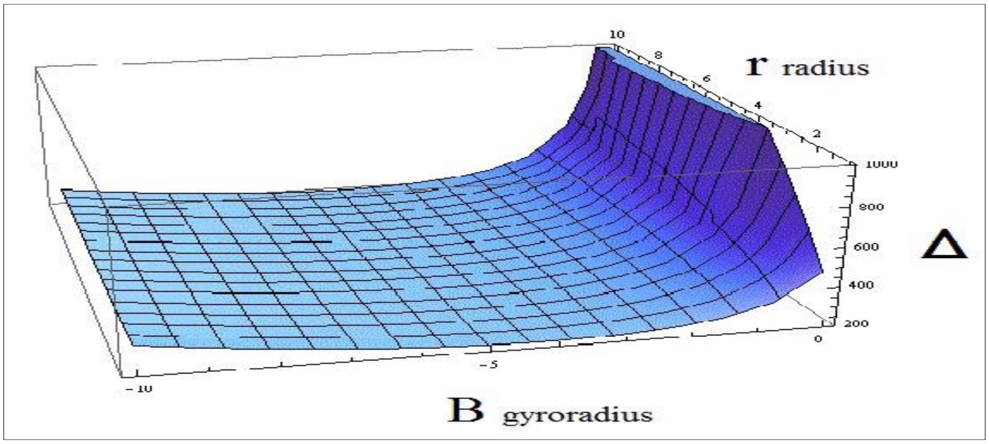

Fig.5 Graph between magnetic field, radius and dimensionless measure of pressure anisotropies.

This is a graph of the algebraic stage of the accelerating cluster dynamo with gravity effect. By increasing the value of radius, the graph goes to infinity. This 3D graph also combine the physical quantities are r radius, B gyroradius and the dimensionless measure of the pressure anisotropies Δ for the algebraic stage of the accelerating cluster dynamo with gravity effect.

All stages of the evolution of the magnetic field described in the previous sections are illustrated in Fig $(1, 2)$ without the gravitational potential Φ and in Fig $(3, 4, 5)$ with gravitational potential which presents the numerical solutions of Eqns $(24),(29),(34)$. We see that numerical results give us the graphs which combine the physical quantities are r radius, B gyroradius and dimensionless measure of the pressure anisotropies Δ. Most of the growth exists over a small fraction of the time during the explosive self-accelerating dynamo stage with the gravity effect. We achieve that, no matter what the seed field is, the unsystematic motion in the Intracluster medium (ICM) will have no difficulty in amplifying it to the observed level in a fraction of the cluster life time.

Acknowledgement: We are thankful to Dr. Babur Majid Mirza, QAU, Islamabad for kind cooperation and overview of this article.

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